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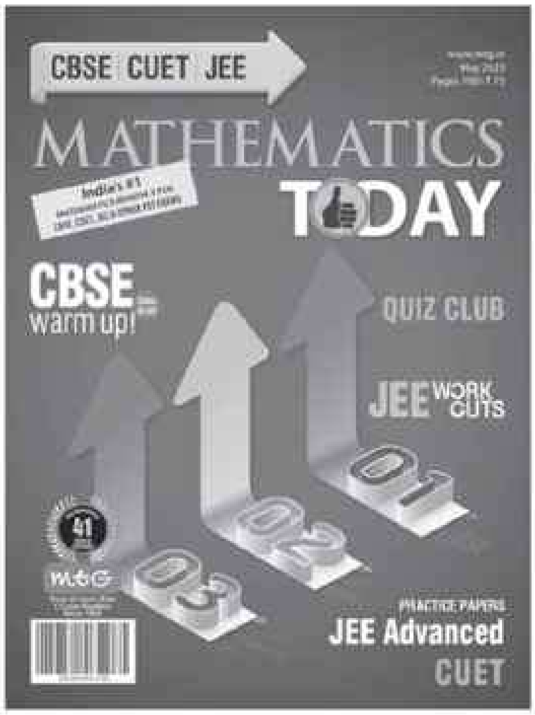

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JEE 2023 PRACTICE PAPER ADVANCED

Exam on 4th June

PAPER-1

SECTION 1 (MAXIMUM MARKS : 24)

- *This section contains EIGHT (08) questions.*
- *The answer to each question is a NUMERICAL VALUE.*
- *If the numerical value has more than two decimal places, truncate/round off the value to TWO decimal places.*
- *Answer to each question will be evaluated according to the following marking scheme:*

Full Marks : +3 ONLY if the correct numerical value is entered;
Zero Marks : 0 In all other cases.

1. If $f(x) = \cos(\log x)$, then

$f\left(\frac{1}{x}\right)f\left(\frac{1}{y}\right)-\frac{1}{2}\left(f\left(\frac{x}{y}\right)+f(xy)\right)$ is equal to ____.

2. For $n \in N$, $n(n + 1)(n + 5)$ is a multiple of ____.

3. If $z+\frac{1}{z}=\sqrt{3}$, then the value of $\sum_{r=1}^{5}\left(z^r+\frac{1}{z^r}\right)^2$ is ____.

4. Number of possible values of n such that $n!$ has exactly 20 zeros at the end is ____.

5. When $19^{93} - 13^{99}$ is divided by 162, the remainder is ____.

6. Let S_k, $k = 1, 2,........, 100$, denote the sum of the infinite geometric series, whose first term $\frac{k-1}{k!}$ and constant ratio is $\frac{1}{k}$. Then the value of

$\frac{100^2}{100!}+\sum_{k=1}^{100}\left|(k^2-3k+1)S_k\right|$ is ____.

7. If $\lim_{x\to 0}\frac{\sin(3x+a)-3\sin(2x+a)+3\sin(x+a)-\sin a}{x^3}$ $= -\cos 1$, then $a =$ ____.

8. Let the point $M(2, 1)$ be shifted through a distance $3\sqrt{2}$ units measured parallel to the line $L: x+y-1=0$ in the direction of decreasing ordinates, to reach at N. If the image of N in the line L is R and d is the distance of R from the line $3x - 4y + 25 = 0$, then the value of $d/2$ is ____.

SECTION 2 (MAXIMUM MARKS : 24)

- *This section contains SIX (06) questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is(are) correct answer(s).*
- *For each question, choose the option(s) corresponding to (all) the correct answer(s).*
- *Answer to each question will be evaluated according to the following marking scheme:*

Full Marks : +4 ONLY if (all) the correct option(s) is(are) chosen;
Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;
Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;
Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : –2 In all other cases.

9. The circle $C_1 : x^2 + y^2 = 3$, with centre at O, intersects the parabola $x^2 = 2y$ at the point P in the first quadrant. Let the tangent to the circle C_1 at P touches other two circles C_2 and C_3 at R_2 and R_3, respectively. Suppose C_2 and C_3 have equal radii $2\sqrt{3}$ and centres Q_2 and Q_3, respectively. If Q_2 and Q_3 lie on the y-axis, then

(a) $Q_2Q_3 = 12$
(b) $R_2R_3 = 4\sqrt{6}$
(c) area of the triangle OR_2R_3 is $6\sqrt{2}$
(d) area of the triangle PQ_2Q_3 is $4\sqrt{2}$.

10. If A and B are two events such that $P(A) = \frac{3}{4}$ and $P(B) = \frac{5}{8}$, then

(a) $P(A \cup B) \geq \frac{3}{4}$ (b) $P(A' \cap B) \leq \frac{1}{4}$

(c) $\frac{3}{8} \leq P(A \cap B) \leq \frac{5}{8}$

(d) none of these

11. A relation R on the set of non-zero complex numbers is defined by $z_1 \, R \, z_2$ if and only if $\frac{z_1 - z_2}{z_1 + z_2}$ is real. Then R is

(a) equivalence (b) reflexive

(c) symmetric (d) none of these

12. Let x_1, x_2, x_3, x_4 be four non-zero numbers satisfying the equation $\tan^{-1}\frac{a}{x} + \tan^{-1}\frac{b}{x} + \tan^{-1}\frac{c}{x} + \tan^{-1}\frac{d}{x} = \frac{\pi}{2}$. Then

(a) $x_1 + x_2 + x_3 + x_4 = a + b + c + d$

(b) $\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \frac{1}{x_4} \neq 0$

(c) $x_1 x_2 x_3 x_4 = abcd$

(d) $(x_2 + x_3 + x_4)(x_3 + x_4 + x_1)(x_4 + x_1 + x_2)(x_1 + x_2 + x_3) = abcd$

13. Suppose A and B are two 3×3 invertible matrices such that $(AB)^k = A^k B^k$ for $k = 2008, 2009, 2010$, then

(a) $AB^{-1} A^{-1} = B^{-1}$ (b) $A^{-1} B^{-2} A = B^2$

(c) $AB = BA$ (d) $A^{-2} B A^2 = A^{-1} BA$

14. Let $f: R \to R, g: R \to R$ and $h: R \to R$ be differentiable functions such that $f(x) = x^3 + 3x + 2$, $g(f(x)) = x$ and $h(g(g(x))) = x$ for all $x \in R$. Then

(a) $g'(2) = \frac{1}{15}$ (b) $h'(1) = 666$

(c) $h(0) = 16$ (d) $h(g(3)) = 36$

SECTION 3 (MAXIMUM MARKS : 12)

- ***This section contains FOUR (04) Matching List Sets.***
- ***Each set has ONE Multiple Choice Question.***
- ***Each set has TWO lists: List-I and List-II.***
- ***List-I has Four entries (I), (II), (III) and (IV) and List-II has Five entries (P), (Q), (R), (S) and (T).***
- ***FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.***
- ***Answer to each question will be evaluated according to the following marking scheme :***
  Full Marks : +3 ONLY if the option corresponding to the correct combination is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : –1 In all other cases.

15. The curve $f(x, y) = 0$ lies in the first quadrant. The tangent at any point on it meets the positive x and y axes at A and B and O is the origin.

List-I		List-II	
(I)	$f(x, y) = xy - 1$	(P)	$AB = 1$
(II)	$f(x, y) = x^2 + y^2 - 1$	(Q)	$OA + OB = 1$
(III)	$f(x, y) = \sqrt{x} + \sqrt{y} - 1$	(R)	$OA \cdot OB = 4$
(IV)	$f(x, y) = x^{2/3} + y^{2/3} - 1$	(S)	$\frac{1}{OA} + \frac{1}{OB} = 1$
		(T)	$\frac{1}{OA^2} + \frac{1}{OB^2} = 1$

The correct option is :

(a) (I) → (S); (II) → (R); (III) → (P); (IV) → (T)
(b) (I) → (R); (II) → (T); (III) → (Q); (IV) → (P)
(c) (I) → (P); (II) → (R); (III) → (T); (IV) → (S)
(d) (I) → (Q); (II) → (R); (III) → (R); (IV) → (S)

16. Match the following.

List-I		List-II	
(I)	$\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{\sqrt{2rn - r^2}} =$	(P)	$\frac{\pi}{6}$
(II)	$\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n} \sin\frac{r\pi}{2n} =$	(Q)	$\frac{\pi}{5}$
(III)	$\lim_{n\to\infty} \sum_{r=0}^{n-1} \frac{1}{\sqrt{4n^2 - r^2}} =$	(R)	$\frac{2}{\pi}$
(IV)	$\lim_{n\to\infty} \sum_{r=1}^{n} \frac{n}{(n+r)\sqrt{r(2n+r)}} =$	(S)	$\frac{\pi}{2}$
		(T)	$\frac{\pi}{3}$

The correct option is :

(a) (I) → (S); (II) → (R); (III) → (P); (IV) → (T)
(b) (I) → (S); (II) → (T); (III) → (Q); (IV) → (P)
(c) (I) → (Q); (II) → (P); (III) → (R); (IV) → (S)
(d) (I) → (P); (II) → (Q); (III) → (P); (IV) → (T)

17. Match the following.

	List-I		List-II
(I)	Area enclosed by $y = \|x\|$, $\|x\| = 1$ and $y = 0$ is	(P)	2
(II)	Area enclosed by $y = \sin x$, $x = 0$, $x = \pi$ and $y = 0$ is	(Q)	–1
(III)	If the area bounded by $x^2 \le y$ and $y \le x + 2$ is $k/4$, then $k =$	(R)	4
(IV)	$\frac{dy}{dx}+\frac{2y}{x}=0, y(1)=1$, then $16y(2) =$	(S)	18
		(T)	1

The correct option is :

(a) (I) → (T); (II) → (P); (III) → (S); (IV) → (R)
(b) (I) → (R); (II) → (T); (III) → (P); (IV) → (P)
(c) (I) → (T); (II) → (P); (III) → (S); (IV) → (R)
(d) (I) → (Q); (II) → (P); (III) → (P); (IV) → (T)

18. Match the following:

	List-I		List-II
(I)	Solution of the differential equation $\left(\frac{dy}{dx}\right)^2-\frac{dy}{dx}(e^x+e^{-x})+1=0$ is given by	(P)	$x + y + 2 = ce^y$, (where c is an arbitrary) constant)
(II)	Solutions of the differential equation $(x + y + 1)dy = dx$ are given by	(Q)	$y = e^x + cx$ (where c is an arbitrary constant)
(III)	Solution of the differential equation $(x + y + 2)dx + (2x + 2y - 1)dy = 0$ are	(R)	$2y + e^{-x} = c$ (where c is an arbitrary constant)
(IV)	Solution of $x\frac{dy}{dx}-y=(x-1)e^x$ is	(S)	$2(x + y + 2) + 5\ln(x + y - 3) = x + c$, (where c is an arbitrary constant)
		(T)	$y - e^x = c'$ (where c is an arbitrary constant)

The correct option is :

(a) (I) → (T); (II) → (S); (III) → (Q); (IV) → (P)
(b) (I) → (S); (II) → (P); (III) → (T); (IV) → (Q)
(c) (I) → (P); (II) → (Q); (III) → (R); (IV) → (S)
(d) (I) → (T); (II) → (P); (III) → (S); (IV) → (Q)

PAPER-2

SECTION-I (MAXIMUM MARKS: 24)

- *This section contains EIGHT (08) questions.*
- *The answer to each question is a SINGLE DIGIT INTEGER ranging from 0 TO 9, BOTH INCLUSIVE.*
- *Answer to each question will be evaluated according to the following marking scheme:*
  Full Marks : +3 If ONLY the correct integer is entered;
  Zero Marks : 0 If the question is unanswered;
  Negative Marks : –1 In all other cases.

1. The number of all values of θ, where $0 < \theta < \pi$, for which the system of equations $(y + z) \cos 3\theta = xyz \sin 3\theta$; $x\sin 3\theta=\frac{2\cos 3\theta}{y}+\frac{2\sin 3\theta}{z}$ and $xyz \sin 3\theta = (y + 2z) \cos 3\theta + y \sin 3\theta$ has a solution (x_0, y_0, z_0) with $y_0z_0 \ne 0$ is ________.

2. Let $y'(x) + y(x)g'(x) = g(x)g'(x)$, $y(0) = 0$, $x \in R$, where $f'(x)$ denotes $\frac{d}{dx}f(x)$ and $g(x)$ is a given non-constant differentiable function on R with $g(0) = g(2) = 0$. Then the value of $y(2)$ is ________.

3. Let $T > 0$ be a fixed real number. Suppose f is a continuous function such that for all $x \in R$,

$f(x + T) = f(x)$. If $I_1=\int_0^T f(x)\,dx$, and

$$I_2=\int_3^{3+3T} f(2x)\,dx.$$

Then the value of $\frac{I_2}{I_1}$ is ________.

4. The number of real roots of the equation $(15+4\sqrt{14})^t+(15-4\sqrt{14})^t=30$, where $t=x^2-4|x|$ is ________.

5. If $\lim_{x\to 0}\frac{(1+x)^{1/x}-e+\frac{ex}{2}}{ex^2}=\frac{m}{n}$, where m and n are relatively prime positive integers, then $3m - n$ is ________.

6. Let A be a 3×3 matrix and if for every column vector X, $X'AX = O$ and if $a_{12} = 145$, $a_{23} = -2008$, $a_{31} = -182$, then $a_{32} + 11\,a_{31}$ must be equal to________.

7. The eccentricity of an ellipse whose axes are the coordinate axes is $\frac{3}{5}$. If the length of its latus rectum is $\frac{32}{5}$, then the maximum distance of its centre from a normal is ________.

8. A vertical line passing through the point $(h, 0)$ intersects the ellipse $\frac{x^2}{4}+\frac{y^2}{3}=1$ at the points P and Q. Let the tangents to the ellipse at P and Q meet at the point R. If $\Delta(h)$ = area of the triangle PQR, $\Delta_1 = \max_{1/2 \le h \le 1} \Delta(h)$ and $\Delta_2 = \min_{1/2 \le h \le 1} \Delta(h)$, then $\frac{8}{\sqrt{5}}\Delta_1 - 8\Delta_2$ is ________.

SECTION-2 (MAXIMUM MARKS : 24)

- ***This section contains SIX (06) questions.***
- ***Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer (s).***
- ***For each question, choose the option(s) corresponding to (all) the correct answer(s).***
- ***Answer to each question will be evaluated according to the following marking scheme:***
  Full Marks : +4 ONLY if (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;
  Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
  Zero Marks : 0 If unanswered;
  Negative Marks: –2 In all other cases.

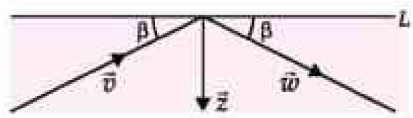
9. Let 'P' be an interior point of ΔABC. If $\angle A = 45°$, $\angle B = 60°$ and $\angle C = 75°$. If X = area of ΔPBC, Y = area of ΔPAC and Z = area of ΔPAB, then which of the following ratios is/are true?
   (a) If P is the centroid, then $X : Y : Z$ is $1 : 1 : 1$.
   (b) If P is the incentre, then $X : Y : Z$ is $2 : \sqrt{6} : (\sqrt{3}+1)$.
   (c) If P is the orthocentre, then $X : Y : Z$ is $1 : \sqrt{3} : (2+\sqrt{3})$.
   (d) If P is the circumcentre, then $X : Y : Z$ is $2 : \sqrt{3} : 1$.

10. If a, b, c are positive rational numbers such that $a > b > c$ and the quadratic equation $(a + b - 2c)x^2 + (b + c - 2a)x + (c + a - 2b)$ has a root in the interval $(-1, 0)$, then
   (a) $c + a < 2b$
   (b) both roots of the given equation are rational.
   (c) the equation $ax^2 + 2bx + c = 0$ has both negative real roots.
   (d) the equation $cx^2 + 2ax + b = 0$ has both negative real roots.

11. If a complex number z satisfy $|z| = 1$ and $\text{amp}(z - 1) = \frac{2\pi}{3}$, then
   (a) $\text{amp}(z^2 + z) = \frac{\pi}{2}$ (b) $z = -\omega^2$
   (c) $z = -\omega$ (d) $|z - 1| = 1$

12. A tangent is drawn at any point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If this tangent is intersected by the tangents at the vertices at points P and Q, then which of the following is/are true
   (a) S, S', P and Q are concyclic
   (b) PQ is diameter of the circle
   (c) S, S', P and Q form rhombus
   (d) PQ is diagonal of acute angle of the rhombus formed by S, S', P and Q

13. The figure shows non-zero vectors $\vec{v}, \vec{w}$ and $\vec{z}$ with $\vec{z}$ orthogonal to the line L, and $\vec{v}$ and $\vec{w}$ making equal angles β with the line L. Assuming $|\vec{v}| = |\vec{w}|$, if the vector $\vec{w}$ is expressed as a linear combination of $\vec{v}$ and $\vec{z}$ as $\vec{w} = x\vec{v} + y\vec{z}$ then



   (a) $x = 1$ (b) $x = \frac{v\sin\beta}{z}$
   (c) $y = 2$ (d) $y = \frac{2v\sin\beta}{z}$

14. A solution curve of the differential equation $(x^2+xy+4x+2y+4)\frac{dy}{dx}-y^2=0, x>0$ passes through the point $(1, 3)$. Then the solution curve
   (a) intersects $y = x + 2$ exactly at one point
   (b) intersects $y = x + 2$ exactly at two points
   (c) intersects $y = (x + 2)^2$
   (d) does not intersect $y = (x + 3)^2$

SECTION-3 (MAXIMUM MARKS : 12)

- ***This section contains FOUR (04) questions.***
- ***Each question has FOUR options (a), (b), (c) and (d). ONLY ONE of these four options is the correct answer.***
- ***For each question, choose the option corresponding to the correct answer.***

- ***Answer to each question will be evaluated according to the following marking scheme.***
Full Marks : +3 If ONLY the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e., the question is unanswered);
Negative Marks : –1 In all other cases.

15. A coin is tossed $2n$ times. The chance that the number of times one gets head is not equal to the number of times one gets tail is

(a) $\frac{(2n)!}{(n!)^2}\cdot\left(\frac{1}{2}\right)^{2n}$ (b) $1-\frac{(2n)!}{(n!)^2}$

(c) $1-\frac{(2n)!}{(n!)^2}\cdot\frac{1}{4^n}$ (d) none of these

16. Let A and B be two matrices such that $A = [a_{ij}]_{2\times3}$

$$=\begin{bmatrix} x & 1 & 2 \\ 3x & -1 & \frac{1}{4x^2+1}\end{bmatrix} \text{and } B=[b_{ij}]_{3\times2}=\begin{bmatrix} \frac{1}{x^2} & \frac{1}{x} \\ 2x & 2 \\ 3 & x\end{bmatrix}$$

where $x > 0$. If matrix C is defined as $C = [c_{ij}]_{2\times2}$, where $c_{ij}=\sum_{r=1}^{3} a_{ir}b_{rj}\,\forall 1\le i, j\le 2$, then the minimum value of $\Delta(x) = \sum_{1\le i\le j\le 2} c_{ij}$ is

(a) 57/4 (b) 57/2
(c) 55/4 (d) 55/2

17. An urn contains 10 balls coloured either black or red. When selecting two balls from the urn at random, the probability that a ball of each colour is selected is 8/15. Assuming that the urn contains more black balls than red balls, the probability that atleast one black ball is selected, when selecting two balls, is :

(a) 18/45 (b) 30/45
(c) 39/45 (d) 41/45

18. The value of $\lim_{n\to\infty}\left(\frac{n!}{(mn)^n}\right)^{1/n}$ is

(a) em (b) e/m
(c) $1/em$ (d) none of these

SOLUTIONS

PAPER-1

1. (0) : We have, $f(x) = \cos(\log x)$

$$f\left(\frac{1}{x}\right)f\left(\frac{1}{y}\right)-\frac{1}{2}\left(f\left(\frac{x}{y}\right)+f(xy)\right)$$

$$=\cos\left(\log\frac{1}{x}\right)\cos\left(\log\frac{1}{y}\right)-\frac{1}{2}\left(\cos\left(\log\frac{x}{y}\right)+\cos(\log xy)\right)$$

$$=\cos\left(\log\frac{1}{x}\right)\cos\left(\log\frac{1}{y}\right)$$

$$-\frac{1}{2}\times2\cos\left(\frac{\log\frac{x}{y}+\log xy}{2}\right)\cos\left(\frac{\log\frac{x}{y}-\log(xy)}{2}\right)$$

$$=\cos\left(\log\frac{1}{x}\right)\cos\left(\log\frac{1}{y}\right)$$

$$-\left(\cos\frac{(\log x-\log y+\log x+\log y)}{2}\right)\times\left(\cos\frac{(\log x-\log y-\log x-\log y)}{2}\right)$$

$$=\cos\left(\log\frac{1}{x}\right)\cos\left(\log\frac{1}{y}\right)-\cos(\log x)\cos(\log y)$$

$= \cos(\log 1-\log x)\cos(\log 1-\log y)-\cos(\log x)\cos(\log y)$.

$= \cos(\log x)\cos(\log y)-\cos(\log x)\cos(\log y)$

($\because \log 1 = 0$)

2. (3) : Let $P(n) : n(n + 1)(n + 5)$

For $n = 1$, $P(1) : 1(1 + 1)(1 + 5) = 12$ which is a multiple of 2 or 3 or 4 or 6 or 12

Now let $P(k)$ for $n = k$ is a multiple of 2 or 3 or 4 or 6 or 12 *i.e.*, $P(k) : k(k + 1)(k + 5)$ is a multiple of 2 or 3 or 4 or 6 or 12 ...(i)

Now let $n = k + 1$,

i.e. $P(k + 1) : (k + 1)(k + 1 + 1)(k + 5 + 1)$

$= k(k + 1)(k + 5) + 2k^2 + 5k + 2k + k^2 + 6k + 5 + 2k + 7$

$= k(k + 1)(k + 5) + 3k^2 + 15k + 12$

$= k(k + 1)(k + 5) + 3(k^2 + 5k + 4)$

$\because$ $3(k^2 + 5k + 4)$ is a multiple of 3 while $k(k + 1)(k + 5)$ is a multiple of 2 or 3 or 4 or 6 or 12 (assumption (i)). So their sum is only multiple of that number which is common in both *i.e.*, 3. So 3 is only multiple of $n(n + 1)(n + 5)$.

3. (8) : $z+\frac{1}{z}=\sqrt{3}$, $z^2-\sqrt{3}z+1=0$

Solving, $z = \text{cis}\left(\pm\frac{\pi}{6}\right)$

$$z^r = \text{cis}\left(\pm\frac{r\pi}{6}\right), \frac{1}{z^r} = \text{cis}\left(\mp\frac{r\pi}{6}\right)$$

$$\therefore \; z^r + \frac{1}{z^r} = 2\cos\frac{r\pi}{6}$$

$$\left(z^r + \frac{1}{z^r}\right)^2 = 4\cos^2\frac{r\pi}{6} = 2\left(1+\cos\frac{r\pi}{3}\right)$$

$$\therefore \; \sum_{r=1}^{5}\left(z^r + \frac{1}{z^r}\right)^2 = 2\sum_{r=1}^{5}\left(1+\cos\frac{r\pi}{3}\right)$$

$$= 2\left[1+\cos\frac{\pi}{3}+1+\cos\frac{2\pi}{3}+1+\cos\pi+1 + \cos\frac{4\pi}{3}+1+\cos\frac{5\pi}{3}\right] = 2(5-1) = 8$$

4. (5) : In any number, number of zeros is equal to exponent of 5.

Number of zeros in $n! = 20$

$$\Rightarrow \; 20 = \sum_{i=1}^{\infty}\left[\frac{n}{5^i}\right] \Rightarrow 20 < \sum_{i=1}^{\infty}\frac{n}{5^i} \quad (\text{as } [x] \le x)$$

$$\Rightarrow \; 20 < n\left(\frac{1}{5}+\frac{1}{5^2}+\frac{1}{5^3}+\ldots\right) \Rightarrow 20 < n\left(\frac{1/5}{1-(1/5)}\right)$$

$$\Rightarrow \; 20 < \frac{n}{4} \Rightarrow n > 80$$

If $n = 80$, exponent of 5 is $\left[\frac{80}{5}\right]+\left[\frac{80}{25}\right] = 19$

If $n = 85$, exponent of 5 is $\left[\frac{85}{5}\right]+\left[\frac{85}{25}\right] = 20$

If $n = 90$, exponent of 5 is $\left[\frac{90}{5}\right]+\left[\frac{90}{25}\right] = 21$

$\Rightarrow \; n \in \{85, 86, 87, 88, 89\}$

5. (0) : $19^{93} = (18+1)^{93}$

$= 1 + \binom{93}{1}\cdot 18$ + multiple of 18^2

$= 1 + 93 \times 18$ + multiple of 9^2

$= 1 + (90 + 3)\,18$ + multiple of 9^2

$= 1 + 54$ + multiple of $9^2 = 55$ + multiple of 81

and $13^{99} = (9+4)^{99} = 4^{99} + \binom{99}{1}9\cdot 4^{98} + \ldots$

$= 4^{99}$ + multiple of $9^2 = 2^{198}$ + multiple of 9^2

$= 8^{66}$ + multiple of $9^2 = (9-1)^{66}$ + multiple of 9^2

$= -66 \times 9 + 1$ + multiple of 9^2

$= -(72 - 6)9 + 1$ + multiple of 9^2

$= 55$ + multiple of 81.

$\therefore$ $19^{93} - 13^{99}$ is a multiple of 81. It is also an even number and hence is a multiple of $81 \times 2 = 162$. The desired remainder is zero.

6. (3) : $S_1 = 0 + 0 + 0 + \ldots\ldots = 0$;

$$S_2 = \frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\ldots = 1$$

$$S_3 = \frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots\ldots = \frac{1}{2};$$

$$S_k = \frac{k-1}{k!} \div \left(1-\frac{1}{k}\right) = \frac{1}{(k-1)!}$$

$$\sum_{k=1}^{3}\left|(k^2-3k+1)S_k\right| = 0 + 1 + \frac{1}{2} = \frac{3}{2}$$

$$\sum_{k=1}^{100}\left|(k^2-3k+1)S_k\right| = \frac{3}{2} + \sum_{k=4}^{100}\frac{((k-1)(k-2)-1)}{(k-1)!}$$

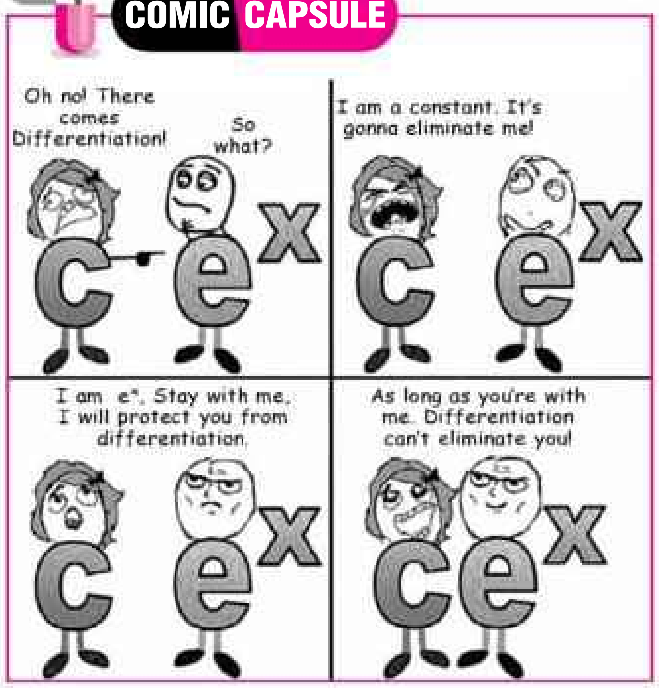
$$= \frac{3}{2} + \sum_{k=4}^{100}\left(\frac{1}{(k-3)!}-\frac{1}{(k-1)!}\right)$$

$$= \frac{3}{2} + \left(\frac{1}{1!}-\frac{1}{3!}\right)+\left(\frac{1}{2!}-\frac{1}{4!}\right)+\left(\frac{1}{3!}-\frac{1}{5!}\right) + \ldots + \left(\frac{1}{96!}-\frac{1}{98!}\right)+\left(\frac{1}{97!}-\frac{1}{99!}\right)$$

$$= \frac{3}{2} + 1 + \frac{1}{2} - \left(\frac{1}{98!}+\frac{1}{99!}\right) = 3 - \frac{100}{99!} = 3 - \frac{100^2}{100!}$$

$\therefore$ The value of the given series is 3.

COMIC CAPSULE



7. (1) : $\displaystyle \lim_{x\to 0}\frac{\sin(3x+a)-3\sin(2x+a)+3\sin(x+a)-\sin a}{x^3}$

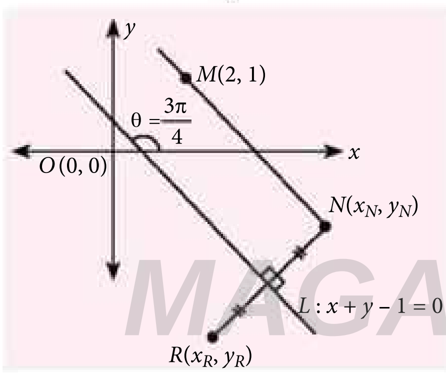
$$=\lim_{x\to 0}\frac{2\sin\frac{3x}{2}\cos\left(\frac{3x+2a}{2}\right)-3\left(2\sin\frac{x}{2}\cos\frac{3x+2a}{2}\right)}{x^3}$$

$$=\lim_{x\to 0}\frac{2\cos\left(\frac{3x+2a}{2}\right)\left[3\sin\frac{x}{2}-4\sin^3\frac{x}{2}-3\sin\frac{x}{2}\right]}{x^3}$$

$$=\lim_{x\to 0}2\cos\left(\frac{3x+2a}{2}\right)\left\{-4\frac{(x/2)^3}{x^3}\right\}\quad \therefore \lim_{\theta\to 0}\sin\theta=\theta$$

$=2\cos a\left(-\frac{4}{8}\right)$ = – cos a = – cos 1 ∴ a = 1

8. (5) : We have $MN = 3\sqrt{2}$



$\therefore\ x_N=2-3\sqrt{2}\left(\frac{-1}{\sqrt{2}}\right)=5$

and $y_n=1-3\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)=-2$

So, coordinates of N are $N(5, -2)$

Image of N in $x + y - 1 = 0$ is $(3, -4)$

∴ Coordinates of R are $R(3, -4)$

So, distance of $R(3, -4)$ from the line $3x - 4y + 25 = 0$ is,

distance = $\dfrac{|3(3)-4(-4)+25|}{\sqrt{(3)^2+(-4)^2}}=\dfrac{50}{5}=10$

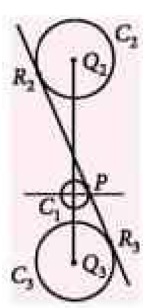
9. (a, b, c) : The point P is given by solving $x^2 + y^2 = 3$ with $x^2 = 2y$ *i.e.*,

$y^2 + 2y = 3 \Rightarrow y^2 + 2y - 3 = 0$

$\Rightarrow\ (y - 1)(y + 3) = 0$

∵ P is in the first quadrant, so $y = 1$

∴ $P(\sqrt{2}, 1)$



The points Q_2 and Q_3 are given by

$Q_2 = (0, 9)$ and $Q_3 = (0, -3)$

$\therefore\ Q_2Q_3=\sqrt{0^2+12^2}=12$

Again, $R_2R_3=\sqrt{(Q_2Q_3)^2-(r_2+r_3)^2}\quad (r_2=r_3=2\sqrt{3})$

$=\sqrt{12^2-(4\sqrt{3})^2}=\sqrt{144-48}=\sqrt{96}=4\sqrt{6}$

The distance of origin from $R_2R_3=\sqrt{3}$

∴ Area of $\Delta OR_2R_3=\frac{1}{2}\cdot 4\sqrt{6}\cdot\sqrt{3}=6\sqrt{2}$ sq. units

Again the distance P from Q_2Q_3 = $\sqrt{2}$ units

$PQ_2Q_3=\frac{1}{2}\cdot 12\cdot\sqrt{2}=6\sqrt{2}$ sq. units

10. (a,b,c) : ∵ $A \subseteq A \cup B$

$\Rightarrow\ P(A) \cap P(A \cup B) \Rightarrow P(A \cup B) \ge \frac{3}{4}$

Also, $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$\ge P(A) + P(B) - 1 = \frac{3}{4}+\frac{5}{8}-1=\frac{3}{8}$

Now, $A \cap B \subseteq B$

$\Rightarrow\ P(A\cap B)\le P(B)=\frac{5}{8}\quad\therefore\quad \frac{3}{8}\le P(A\cap B)\le\frac{5}{8}\quad \ldots\text{(i)}$

and $P(A \cap B') = P(A) - P(A \cap B)$

$\Rightarrow\ \frac{3}{4}-\frac{5}{8}\le P(A\cap B')\le\frac{3}{4}-\frac{3}{8}\ \Rightarrow\ \frac{1}{8}\le P(A\cap B')\le\frac{3}{8}$

$\therefore\ P(A \cap B) = P(B) - P(A' \cap B)$ [using eq.(i)]

$\Rightarrow\ \frac{3}{8}\le P(B)-P(A'\cap B)\le\frac{5}{8}\ \Rightarrow\ 0\le P(A'\cap B)\le\frac{1}{4}$

11. (a,b,c) : Clearly $z\,R\,z$ as $\frac{z-z}{z+z}=0$ is real number.

Hence, R is reflexive. Also, R is symmetric as if $\frac{z_1-z_2}{z_1+z_2}$ is a real number then $\frac{z_2-z_1}{z_1+z_2}$ is also a real number.

If $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$, then $z_1\,Rz_2$ iff

$\frac{(x_1-x_2)+i(y_1-y_2)}{(x_1+x_2)+i(y_1+y_2)}$ is a real number

$\Leftrightarrow\ \frac{((x_1-x_2)+i(y_1-y_2))((x_1+x_2)-i(y_1+y_2))}{(x_1+x_2)^2+(y_1+y_2)^2}$

is a real number

$\Leftrightarrow\ (y_1 - y_2)(x_1 + x_2) - (x_1 - x_2)(y_1 + y_2) = 0$

$\Leftrightarrow\ x_1/y_1 = x_2/y_2.$

Similarly for $z_2 = x_2 + iy_2$ and $z_3 = x_3 + iy_3$ then $z_2\,Rz_3$ iff $x_2/y_2 = x_3/y_3$

If $z_1\,R\,z_2$ and $z_2\,R\,z_3$ then $x_1/y_1 = x_2/y_2 = x_3/y_3$.

$\frac{x_1}{y_1}=\frac{x_3}{y_3}\Rightarrow z_1\,R\,z_3$. So, R is transitive.

Hence, R is an equivalence relation.

12. (c,d): $\tan^{-1}\frac{a}{x}+\tan^{-1}\frac{b}{x}=\frac{\pi}{2}-\left(\tan^{-1}\frac{c}{x}+\tan^{-1}\frac{d}{x}\right)$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{a}{x}+\frac{b}{x}}{1-\frac{ab}{x^2}}\right)=\frac{\pi}{2}-\tan^{-1}\left(\frac{\frac{c}{x}+\frac{d}{x}}{1-\frac{cd}{x^2}}\right)$$

$$\Rightarrow \tan^{-1}\frac{(a+b)x}{x^2-ab}=\cot^{-1}\frac{(c+d)x}{x^2-cd}$$

$$\Rightarrow \frac{(a+b)x}{x^2-ab}=\frac{x^2-cd}{(c+d)x}$$

$$\Rightarrow x^4-(ab+ac+ad+bc+bd+cd)x^2+abcd=0$$

Its roots are x_1, x_2, x_3, x_4

$\therefore \sum x_1 = 0, \sum x_1x_2 = -\sum ab, \sum x_1x_2x_3 = 0$

and $x_1x_2x_3x_4 = abcd$

$\therefore \sum\frac{1}{x_1} = 0$ and

$(x_2 + x_3 + x_4)(x_3 + x_4 + x_1)(x_4 + x_1 + x_2)(x_1 + x_2 + x_3)$
$= x_1x_2x_3x_4 = abcd$

13. (a,c,d) : Since, A and B are invertible matrices, then A^{-1} and B^{-1} both exist. Also, for every positive integer, A^n and B^n are invertible.

Now, $(AB)^k = A^k B^k$ hold for $k = 2008, 2009, 2010$.

Now, $(AB)^{2008} = A^{2008} B^{2008}$ ...(i)

$(AB)^{2009} = A^{2009} B^{2009}$ ...(ii)

$(AB)^{2010} = A^{2010} B^{2010}$ ...(iii)

From (ii), we get

$A^{2009} B^{2009} = (AB)^{2009} = (AB)^{2008} (AB)$

$\Rightarrow A^{2008} AB^{2008} B = A^{2008} B^{2008} AB$ [Using (i)]

Now, A^{2008} and B are invertible matrices, then from the above equation, we get

$AB^{2008} = B^{2008} A$ ...(iv)

Similarly, from (ii) and (iii), we get

$AB^{2009} = B^{2009} A$ ...(v)

Now, $(AB) B^{2008} = AB^{2009} = B^{2009} A$ [Using (v)]

$= B(B^{2008} A) = B (AB^{2008})$ [Using (iv)]

$= (BA) B^{2008}$

Thus, $(AB) B^{2008} = (BA) B^{2008}$

Since, B^{2008} is an invertible matrix, then $AB = BA$

Now, $AB = BA$

$\Rightarrow (AB)^{-1} = (BA)^{-1} \Rightarrow B^{-1}A^{-1} = A^{-1}B^{-1}$

$\Rightarrow AB^{-1}A^{-1} = (AA^{-1}) B^{-1} \Rightarrow AB^{-1}A^{-1} = IB^{-1}$

$\Rightarrow AB^{-1}A^{-1} = B^{-1}$

Now, $A^{-2}BA^2 = A^{-2}BAA = A^{-2}ABA$

$= A^{-1}A^{-1}ABA = A^{-1}BA$

14. (b,c) : $f(x) = x^3 + 3x + 2$, $g(f(x)) = x$, $h(g(g(x))) = x$

Now, $f'(x) = 3x^2 + 3$

Again, $g'(2)=\frac{1}{f'(0)}=\frac{1}{3}$ (As we have $f(0) = 2$)

Now, $h(g(g(x))) = x$

Differentiating with respect to x,

$$h'(g(g(x)))=\frac{1}{g'(g(x))\,g'(x)}$$

Now, to solve $g(g(x)) = 1$ we have $g(x) = f(1) = 6$
$f(6) = 236$

$$\therefore h'(1)=\frac{1}{g'(6)g'(236)}=\frac{1}{\frac{1}{6}\cdot\frac{1}{111}}=666$$

Solving, $g(g(x)) = 0$ means $g(x) = g^{-1}(0)$.

$\Rightarrow g(x) = 2 \quad \therefore x = g^{-1}(2) = f(2) = 16$

$\therefore h(0) = 16$. Again, $h(g(g(x))) = x$

Put x to $f(x)$ then $h(g(g(f(x)))) = f(x)$

$\Rightarrow h(g(x)) = f(x) \quad \therefore h(g(3)) = f(3) = 38$

15. (b) : (I) Tangent at $\left(t, \frac{1}{t}\right)$ is $y-\frac{1}{t}=\frac{-1}{t^2}(x-t)$

which meets the axes at $A(2t, 0)$ and $B\left(0, \frac{2}{t}\right)$.

$\therefore OA \cdot OB = 2t\cdot\frac{2}{t} = 4$

(II) Tangent at $(\cos\theta, \sin\theta)$ is $x\cos\theta + y\sin\theta = 1$,

which meets the axes at $A\left(\frac{1}{\cos\theta}, 0\right)$ and $B\left(0, \frac{1}{\sin\theta}\right)$

$\therefore \frac{1}{OA^2}+\frac{1}{OB^2} = \cos^2\theta + \sin^2\theta = 1$

(III) The tangent at $(\cos^4\theta, \sin^4\theta)$ is

$$y - \sin^4\theta = -\frac{\sin^2\theta}{\cos^2\theta}(x - \cos^4\theta)$$

which meets the axes at $A(\cos^2\theta, 0)$ and $B(0, \sin^2\theta)$

$\therefore OA + OB = \cos^2\theta + \sin^2\theta = 1$

(IV) The tangent at $(\cos^3\theta, \sin^3\theta)$ is

$$y - \sin^3\theta = -\frac{\sin\theta}{\cos\theta}(x - \cos^3\theta)$$

which meets the axes at $A(\cos\theta, 0)$ and $B(0, \sin\theta)$.

$\therefore AB=\sqrt{\cos^2\theta+\sin^2\theta}=1$

16. (a) : (I) $\lim_{n\to\infty}\frac{1}{n}\sum\frac{1}{\sqrt{\frac{2r}{n}-\left(\frac{r}{n}\right)^2}}$

$$=\int_0^1\frac{dx}{\sqrt{2x-x^2}}=\int_0^1\frac{dx}{\sqrt{1-(1-x)^2}}$$

$$=\int_0^1\frac{dx}{\sqrt{1-x^2}}=\left[\sin^{-1}x\right]_0^1=\frac{\pi}{2}$$

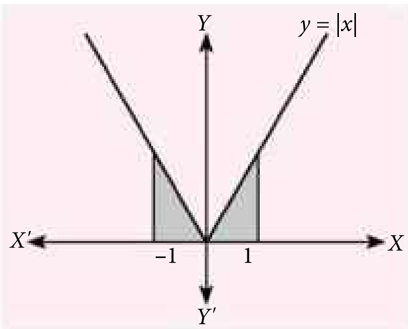
(II) $\int_0^1 \sin\frac{\pi x}{2}dx = \left[\frac{-2}{\pi}\cos\frac{\pi x}{2}\right]_0^1 = \frac{2}{\pi}$

(III) $\int_0^1 \frac{dx}{\sqrt{4-x^2}} = \left[\sin^{-1}\frac{x}{2}\right]_0^1 = \frac{\pi}{6}$

(IV) $\int_0^1 \frac{dx}{(1+x)\sqrt{2x+x^2}} = \int_0^1 \frac{dx}{(1+x)\sqrt{(1+x)^2-1}}$,

$= \sec^{-1}(1+x)\Big|_0^1 = \sec^{-1}2 - \sec^{-1}1 = \frac{\pi}{3} - 0 = \frac{\pi}{3}$

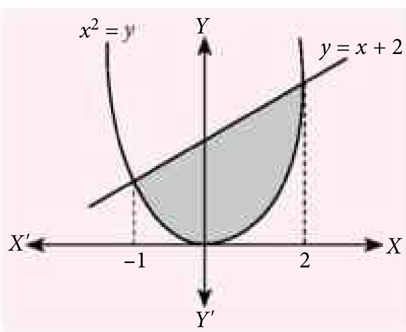
17. (c) : (I)



$A = 2\int_0^1 x dx = 2\left[\frac{x^2}{2}\right]_0^1 = 1$ sq. unit

(II) $A = \int_0^{\pi} |\sin x|\, dx = \int_0^{\pi} \sin x = [-\cos x]_0^{\pi} = 2$

(III)



$A = \int_1^2 \left[(x+2) - x^2\right]dx = \left(\frac{x^2}{2} + 2x - \frac{x^3}{3}\right)_{-1}^2 = \frac{9}{2}$

$\therefore \frac{k}{4} = \frac{9}{2} \Rightarrow k = 18$

(IV) $\frac{dy}{y} = -2\frac{dx}{x} \Rightarrow y = cx^{-2}$

At $y(1) = 1 \Rightarrow 1 = c(1)^{-2} \Rightarrow c = 1$

$y = x^{-2}$

$\therefore \quad y(2) = (2)^{-2} = \frac{1}{4}$

$\therefore \quad 16y(2) = 16 \times \frac{1}{4} = 4$

18. (d) : (I) The given equation can be written as

$\left(\frac{dy}{dx} - e^{-x}\right)\left(\frac{dy}{dx} - e^{x}\right) = 0$

$\Rightarrow \frac{dy}{dx} - e^{-x} = 0$ or $\frac{dy}{dx} - e^{x} = 0$

$\Rightarrow y + e^{-x} = c$ or $y - e^{x} = c'$

(Where, c and c' are arbitrary constant)

(II) $\because \quad (x + y + 1)dy = dx$

Put $x + y + 1 = v$

$\Rightarrow dx + dy = dv$ and the given equation reduces to $v(dv - dx) = dx \Rightarrow x + c_1 = v - \ln(v + 1)$

$\Rightarrow \ln(v + 1) = v - x - c_1$

or $\ln(x + y + 2) = y + 1 - c_1 = y + c_2$

Also, $x + y + 2 = e^{y+c_2} = e^{y} \cdot e^{c_2} = ce^{y}$

(Where, c is an arbitrary constant)

(III) Given, $(x + y + 2)dx + (2x + 2y - 1)dy = 0$ ...(i)

Put $x + y + 2 = v \Rightarrow 1 + \frac{dy}{dx} = \frac{dv}{dx}$

Then (i) becomes $\frac{dv}{dx} - 1 = \frac{-v}{2v-5}$

$\Rightarrow \frac{dv}{dx} = 1 - \frac{v}{2v-5} = \frac{v-5}{2v-5}$

$\Rightarrow \frac{dv(2v-5)}{v-5} = dx$ ...(ii)

On integrating (ii) we get, $x + c = 2v + 5\ln(v - 5)$

$\Rightarrow x + c = 2(x + y + 2) + 5\ln(x + y - 3)$

(Where, c is an arbitrary constant)

(IV) We have, $\frac{dy}{dx} - \frac{y}{x} = \frac{(x-1)}{x}e^{x}$

It is a linear D.E. with I.F. $= e^{-\int \frac{1}{x}dx} = \frac{1}{x}$

$\therefore$ Solution is $y\left(\frac{1}{x}\right) = \int \frac{(x-1)e^{x}}{x} \times \frac{1}{x}dx$

$\Rightarrow y\left(\frac{1}{x}\right) = \int e^{x}\left(\frac{1}{x} - \frac{1}{x^2}\right)dx$

$\Rightarrow y\left(\frac{1}{x}\right) = \frac{e^{x}}{x} + c \Rightarrow y = e^{x} + cx$,

(where, c is an arbitrary constant).

MONTHLY TEST DRIVE CLASS XI ANSWER KEY

1.	(b)	2.	(c)	3.	(b)	4.	(a)	5.	(c)
6.	(b)	7.	(b,c,d)	8.	(a,b,c)	9.	(a,c)	10.	(a,d)
11.	(b,c)	12.	(a,b)	13.	(a,b)	14.	(a)	15.	(c)
16.	(b)	17.	(1.73)	18.	(1025)	19.	(1)	20.	(78)

PAPER-2

1. (3) : The equations are

$xyz \sin 3\theta = (y + z)\cos 3\theta$ ....(i)

$xyz \sin 3\theta = 2z \cos 3\theta + 2y \sin 3\theta$ ...(ii)

$xyz \sin 3\theta = (y + 2z)\cos 3\theta + y \sin 3\theta$ ...(iii)

(ii) – (iii) $\Rightarrow y(\sin 3\theta - \cos 3\theta) = 0$

$\therefore \tan 3\theta = 1 = \tan\frac{\pi}{4},\ 3\theta = n\pi + \frac{\pi}{4}$

$\Rightarrow \theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{9\pi}{12}.$

2. (0) : Given $y'(x) + y(x)g'(x) = g(x)g'(x),\ y(0) = 0$

Integrating factor $= e^{\int g'(x)dx} = e^{g(x)}$

Solution is $y(x)e^{g(x)} = \int e^{g(x)} g(x)g'(x)dx + k$

Let $g(x) = v$, then $y(x)e^{g(x)} = \int e^v v\,dv + k = ve^v - e^v + k$

Thus $y(x) = (g(x) - 1) + ke^{-g(x)}$

$x = 0,\ y(0) = (g(0) - 1) + ke^{-g(0)}$ giving $k = 1$

$y(2) = (g(2) - 1) + e^{-g(2)}$. Thus $y(2) = 0$

3. (3) : $I_2 = \int_3^{3+3T} f(2x)dx$, put $2x = t \Rightarrow 2dx = dt$

$\therefore I_2 = \frac{1}{2}\int_6^{6+6T} f(t)dt = \frac{1}{2}\int_0^{6T} f(t)dt = \frac{6}{2}\int_0^{T} f(t)dt = 3I_1$

$\Rightarrow \frac{I_2}{I_1} = 3$

4. (6) : $15 - 4\sqrt{14} = \left(15 + 4\sqrt{14}\right)^{-1}$

If $y = \left(15 + 4\sqrt{14}\right)^t$, then $y + \frac{1}{y} = 30$

$\Rightarrow y^2 - 30y + 1 = 0$

$\Rightarrow y = 15 \pm 4\sqrt{14} = \left(15 + 4\sqrt{14}\right)^t$

$\therefore t = \pm 1 \Rightarrow x^2 - 4|x| = \pm 1$

$(|x| - 2)^2 = 3, 5 \Rightarrow |x| = 2 \pm \sqrt{3}, 2 + \sqrt{5}$

$\therefore x = \pm\left(2 - \sqrt{3}\right), \pm\left(2 + \sqrt{3}\right), \pm\left(2 + \sqrt{5}\right)$

5. (9) : $L = \lim_{x\to 0} \frac{e \cdot e^{-\frac{x}{2}+\frac{x^2}{3}\cdots} - e + \frac{ex}{2}}{ex^2}$

$= \lim_{x\to 0} \frac{e^{-\frac{x}{2}+\frac{x^2}{3}\cdots\cdots} - 1 + \frac{x}{2}}{x^2}$

$= \lim_{x\to 0} \frac{1 + \left(-\frac{x}{2} + \frac{x^2}{3}\right) + \frac{\left(-\frac{x}{2} + \frac{x^2}{3}\right)^2}{2} \ldots - 1 + \frac{x}{2}}{x^2}$

$= \frac{1}{3} + \frac{1}{8} = \frac{11}{24} = \frac{m}{n}$

$\therefore 3m - n = 9$

6. (6) : Let $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, then $X' = [x_1\ x_2\ x_3]$

Then, $X'AX = [x_1\ x_2\ x_3]\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique answer.

					9	7											7	1		
	4	6		8			3						2	9		3			6	
3			4	1			9					6			2	5			4	
2			9		4	6						9			6		5	3		
	1	5				9	4						6	7				4	9	
		4	1		3			7						3	7		9			6
	2			7	1			9					1			8	6			9
	6			9		4	7						9			7		8	5	
		9	2							9				8	5					
									3		8									
								3				6								
									7		9									
					3	6				7							2	4		
	7	8		5			3						2	7		5			1	
3			4	6			9					1			9	7			5	
7			5		1	2						3			7		9	5		
	5	9				8	7						9	8				1	7	
		4	8		7			3						5	2		8			3
	3			1	9			5					3			4	1			5
	9			3		4	1						1			2		8	9	
		7	2											2	5					

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

$$=[x_1\ x_2\ x_3]\begin{bmatrix} a_{11}x_1+a_{12}x_2+a_{13}x_3 \\ a_{21}x_1+a_{22}x_2+a_{23}x_3 \\ a_{31}x_1+a_{32}x_2+a_{33}x_3 \end{bmatrix}$$

$$=\begin{bmatrix} a_{11}x_1^2+a_{22}x_2^2+a_{33}x_3^2+(a_{12}+a_{21})x_1x_2 \\ +(a_{23}+a_{32})x_2x_3+(a_{13}+a_{31})x_3x_1 \end{bmatrix}=O \text{ (given)}$$

$\therefore\ a_{11}x_1^2+a_{22}x_2^2+a_{33}x_3^2+(a_{12}+a_{21})x_1x_2$
$+ (a_{23} + a_{32})x_2 x_3 + (a_{13} + a_{31})x_1x_3 = 0$

This is true for every x_1, x_2, x_3, then
$a_{11} = 0$, $a_{22} = 0$, $a_{33} = 0$, $a_{12} + a_{21} = 0$,
$a_{23} + a_{32} = 0$, $a_{13} + a_{31} = 0$
$\therefore\ a_{12} = 145 \Rightarrow a_{21} = -145$
$a_{23} = -2008 \Rightarrow a_{32} = 2008$
$a_{31} = -182 \Rightarrow a_{13} = 182$
Now, $a_{32} + 11\,a_{31} = 2008 - 11 \times 182 = 6$

7. (1) : Length of latus rectum

$$= 2a(1-e^2) = \frac{32}{5} \Rightarrow a\left(1-\frac{9}{25}\right) = \frac{16}{5}$$

$$\therefore\ a = 5,\ b = a\sqrt{1-e^2} = 5\sqrt{1-\frac{9}{25}} = 4$$

The ellipse is $\frac{x^2}{25} + \frac{y^2}{16} = 1$

The normal at $(5\cos\theta, 4\sin\theta)$ is

$$\frac{5x}{\cos\theta} - \frac{4y}{\sin\theta} = 5^2 - 4^2 = 9$$

The distance of the centre from it is

$$d = \frac{9}{\sqrt{\frac{25}{\cos^2\theta}+\frac{16}{\sin^2\theta}}} = \frac{9}{\sqrt{25\sec^2\theta + 16\operatorname{cosec}^2\theta}}$$

$$= \frac{9}{\sqrt{41 + 25\tan^2\theta + 16\cot^2\theta}}$$

But $25\tan^2\theta + 16\cot^2\theta \ge 2 \cdot 5 \cdot 4 = 40$ by A.M.G.M. inequality.

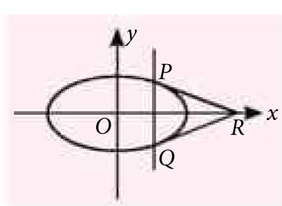
$$\therefore\ d \le \frac{9}{\sqrt{41+40}} = 1.$$

8. (9) : The line $h = 0$ meets the ellipse $\frac{x^2}{4}+\frac{y^2}{3}=1$

at $y = \pm\frac{\sqrt{3}}{2}\sqrt{4-h^2}$

$P\left(h, \frac{\sqrt{3}}{2}\sqrt{4-h^2}\right)$,

$Q\left(h, -\frac{\sqrt{3}}{2}\sqrt{4-h^2}\right)$



Let R be $(\alpha, 0)$. Then PQ is chord of contact for the point $R(\alpha, 0)$.

Compare $x = h$ with $\frac{x\alpha}{4} = 1$ we get $\alpha = \frac{4}{h}$

$$\Delta(h) = \text{area of triangle } PQR = \frac{1}{2}\cdot\frac{2\sqrt{3}}{2}\sqrt{4-h^2}\cdot\left(\frac{4}{h}-h\right)$$

$$= \frac{\sqrt{3}}{2h}(4-h^2)^{3/2}$$

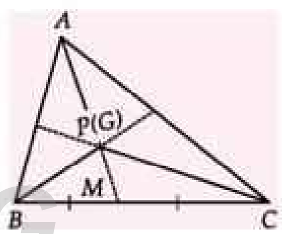
$$\Delta'(h) = -\frac{\sqrt{3}(4+2h^2)}{2h^2}(4-h^2)^{1/2} < 0$$

Thus Δ is decreasing.

$$\therefore\ \Delta_1 = \Delta\left(\frac{1}{2}\right) = \frac{45\sqrt{5}}{8} \text{ and } \Delta_2 = \Delta(1) = \frac{9}{2}$$

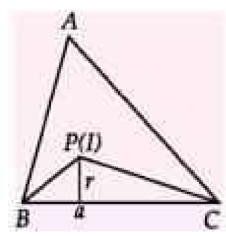
$$\text{Thus, } \frac{8}{\sqrt{5}}\Delta_1 - 8\Delta_2 = \frac{8}{\sqrt{5}}\cdot\frac{45\sqrt{5}}{8} - 8\cdot\frac{9}{2} = 45 - 36 = 9$$

9. (a,b,c,d) : (a) Using properties of median, we have
arΔPBC = arΔPCA
= arΔPAB
$\therefore$ arΔPBC : arΔPCA : arΔPAB = 1 : 1 : 1



(b) arΔPBC : arΔPCA : arΔPAB

$= \frac{1}{2}ar : \frac{1}{2}br : \frac{1}{2}cr$
$= a : b : c$
$= \sin 45° : \sin 60° : \sin 75°$
$= 2 : \sqrt{6} : (\sqrt{3}+1)$

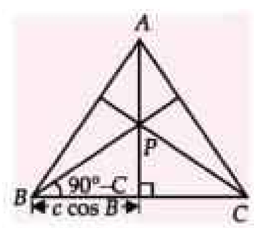


(c) arΔPBC : arΔPCA : arΔPAB

$= \frac{1}{2}a(2R\cos B\cos C)$

$: \frac{1}{2}b(2R\cos C\cos A)$

$: \frac{1}{2}c(2R\cos A\cos B)$



$= \sin A\cos B\cos C : \sin B\cos C\cos A : \sin C\cos A\cos B$

$= \tan 45° : \tan 60° : \tan 75° = 1 : \sqrt{3} : (2+\sqrt{3})$

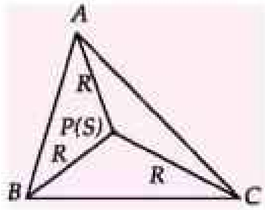
MONTHLY TEST DRIVE CLASS XII ANSWER KEY

1.	(d)	**2.**	(a)	**3.**	(a)	**4.**	(b)	**5.**	(d)
6.	(c)	**7.**	(a, c)	**8.**	(a)	**9.**	(c,d)	**10.**	(d)
11.	(a,c)	**12.**	(a,d)	**13.**	(b,c)	**14.**	(b)	**15.**	(c)
16.	(b)	**17.**	(0.25)	**18.**	(5)	**19.**	(6)	**20.**	(781)

(d) $ar\Delta PBC : ar\Delta PCA : ar\Delta PAB$

$= \frac{1}{2}R^2 \sin 2A : \frac{1}{2}R^2 \sin 2B$

$: \frac{1}{2}R^2 \sin 2C$



$= \sin 2A : \sin 2B : \sin 2C$

$= \sin 90° : \sin 120° : \sin 150° = 2 : \sqrt{3} : 1$

10. (a,b,c,d) : We have, $a > b > c$ ...(i)

and given equation is :

$(a + b - 2c)x^2 + (b + c - 2a)x + (c + a - 2b) = 0$ ...(ii)

Since equation (ii) has a root in the interval (–1, 0), we have $f(-1) f(0) < 0$

$\Rightarrow (2a - b - c)(c + a - 2b) < 0$ ...(iii)

From (i), $a > b \Rightarrow a - b > 0$ and $a > c \Rightarrow a - c > 0$

$\therefore 2a - b - c > 0$ ...(iv)

From (iii) and (iv), $c + a - 2b < 0$ or $c + a < 2b$

Again, Sum of coefficient of the equation = 0

That means one root is 1 and the other root is $\frac{c+a-2b}{a+b-2c}$,

which is a rational number as a, b, c are rational.

Further, the discriminant of equation

$ax^2 + 2bx + c = 0$ is $D = 4b^2 - 4ac$

As deduced earlier, $c + a < 2b \Rightarrow 4b^2 > (c + a)^2$

$\Rightarrow 4b^2 > c^2 + a^2 + 2ac$

$\Rightarrow 4b^2 - 4ac > c^2 + a^2 - 2ac = (c - a)^2 \Rightarrow 4b^2 - 4ac > 0$

$\Rightarrow D > 0$

Also, each of a, b, c is positive.

Therefore, the equation $ax^2 + 2bx + c = 0$ has real and negative roots.

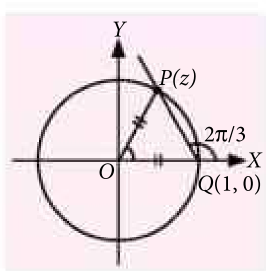
Similarly, $cx^2 + 2ax + b = 0$ has both negative real roots.

11. (a,b,d) : $|z| = 1$ *i.e.*, z lies on a circle whose centre is at origin and radius is 1. $\arg(z - 1) = \frac{2\pi}{3}$

So, z lies on a ray emanating from (1, 0) making an angle of $\frac{2\pi}{3}$ with positive real axis.

$\Rightarrow \angle PQX = \frac{2\pi}{3}$

$\Rightarrow \angle PQO = \frac{\pi}{3}$ and $OP = OQ$



So, triangle OPQ is equilateral

$\Rightarrow OP = OQ = PQ$

$\Rightarrow |z| = |z - 1| = 1$

Now, $\arg(z) = 60°$

$\Rightarrow z = \cos 60° + i \sin 60° = \frac{1+i\sqrt{3}}{2} = -\omega^2$

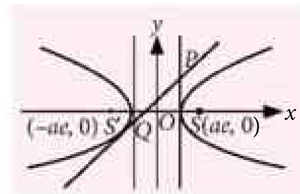
$\Rightarrow z^2 = \omega^4 = \omega$

$\therefore z^2 + z = \omega - \omega^2$

$\left(\frac{-1+i\sqrt{3}}{2}\right) - \left(\frac{-1-i\sqrt{3}}{2}\right) = i\sqrt{3} \Rightarrow \arg(z^2 + z) = \frac{\pi}{2}$

12. (a,b) : Any tangent to the hyperbola is

$\frac{x\sec\theta}{a} - \frac{y\tan\theta}{b} = 1$



Solving this line with the line $x = \pm a$, we get the coordinates of points P and Q as

$\left(a, b\tan\frac{\theta}{2}\right)$ and $\left(-a, -b\cot\frac{\theta}{2}\right)$.

Now slopes of the lines PS and QS are

$m_{PS} = \frac{b\tan\frac{\theta}{2}}{a(1-e)}, m_{QS} = \frac{-b\cot\frac{\theta}{2}}{-a(1+e)}$

$\Rightarrow m_{PS} \cdot m_{QS} = \frac{-b^2}{a^2(e^2-1)} = -1$

Similarly $m_{PS'} \cdot m_{QS'} = -1$

$\Rightarrow$ Line PQ subtends an angle of $\frac{\pi}{2}$ at S and S'

$\Rightarrow$ Points P, Q, S and S' are concyclic.

$\Rightarrow$ PQ is diameter.

13. (a, d) : $|\vec{v}| = |\vec{w}| = v, |\vec{z}| = z$

Given $\vec{w} = x\vec{v} + y\vec{z}$

$\vec{w} \cdot \vec{v} = |\vec{w}| \cdot |\vec{v}| \cos 2\beta = v^2 \cos 2\beta$

$\vec{w} \cdot \vec{z} = |\vec{w}| \cdot |\vec{z}| \cos\left(\frac{\pi}{2} - \beta\right) = vz \sin\beta$

$\vec{v} \cdot \vec{z} = |\vec{v}||\vec{z}| \cos\left(\frac{\pi}{2} + \beta\right) = -vz\sin\beta$

Now $\vec{w} = x\vec{v} + y\vec{z}$

Talking dot product with $\vec{v}$, we get

$\vec{w} \cdot \vec{v} = x\vec{v} \cdot \vec{v} + y\vec{z} \cdot \vec{v}$

$\Rightarrow v^2 \cos 2\beta = v^2 x + y(-vz \sin\beta)$

$\Rightarrow (v)x - (z \sin\beta)y = v \cos 2\beta$ ...(i)

Taking dot product with $\vec{w}$, we get

$\vec{w} \cdot \vec{w} = x\vec{v} \cdot \vec{w} + y\vec{z} \cdot \vec{w}$

$\Rightarrow v^2 = xv^2 \cos 2\beta + y\, vz \sin\beta$

$\Rightarrow (v \cos 2\beta)x + (z \sin\beta)y = v$ ...(ii)

Solving equation (i) and (ii), we have

$x = 1$ and $y = \frac{2v\sin\beta}{z}$

14. (a, d) : Put $x + 2 = u$ and $y = v$ leads to

$$(u^2+uv)\frac{dv}{du}=v^2$$

$\Rightarrow\ (u^2 + uv)dv = v^2du \Rightarrow u^2dv = v(vdu - udv)$

We have $\dfrac{dv}{v}=\dfrac{vdu-udv}{u^2}$

On integrating, we get, $\ln|v|=-\dfrac{v}{u}+\lambda$

As the curve passes through (1, 3), we have
$\lambda = 1 + \ln 3$

Then the curve is $\dfrac{y}{x+2}+\ln|y|-1-\ln 3=0,\ x>0$

Substitute $y = x + 2$ in the equation of curve we have
$1 + \ln|x + 2| - 1 - \ln 3 = 0\ \therefore\ x = 1, -5$
The curve intersects $y = x + 2$ at point (1, 3).
Put $y = (x + 2)^2$ in the equation of the curve to get
$(x + 2) + 2\ln(x + 2) = 1 + \ln 3$
As the L.H.S. is an increasing function, hence it is greater than $2 + 2\ln 2$. Thus no solution.
Now put $y = (x + 3)^2$ in equation of curve,

$$\frac{(x+3)^2}{x+2}+\ln(x+3)^2-1-\ln 3=0$$

As $x > 0$, we have $x + 3 > x + 2$ *i.e.* $x + 3 > 3$

$$\frac{(x+3)^2}{x+2}+\frac{\ln(x+3)^2}{3}>1$$

Hence again there is no solution.
Thus, the curve $y = (x + 3)^2$ doesn't intersect the original curve.

15. (c) : Let p be the probability of getting head when a coin is thrown once and q be the probability of not getting head

$$\therefore\ p=\frac{1}{2}\text{ and } q=1-p=1-\frac{1}{2}=\frac{1}{2}$$

Let X be the random variable denoting the number of heads when a coin is tossed $2n$ times.
Required probability = 1 – Probability of getting equal number of heads and tails = $1 - P(X = n)$

$$=1-{}^{2n}C_n\left(\frac{1}{2}\right)^{2n-n}\left(\frac{1}{2}\right)^{n}=1-{}^{2n}C_n\left(\frac{1}{2}\right)^{n}\cdot\left(\frac{1}{2}\right)^{n}$$

$$=1-{}^{2n}C_n\left(\frac{1}{2}\right)^{2n}=1-\frac{(2n)!}{n!n!}\cdot\left(\frac{1}{4}\right)^{n}=1-\frac{(2n)!}{(n!)^2}\cdot\frac{1}{4^n}$$

16. (a) : $C=AB=\begin{bmatrix} x & 1 & 2 \\ 3x & -1 & \dfrac{1}{4x^2+1}\end{bmatrix}\begin{bmatrix}\dfrac{1}{x^2} & \dfrac{1}{x} \\ 2x & 2 \\ 3 & x\end{bmatrix}$

$$\therefore\ \Delta(x)=\sum_{1\le i\le j\le 2} c_{ij}=c_{11}+c_{12}+c_{22}$$

$$=\frac{1}{x}+2x+6+1+2+2x+3-2+\frac{x}{4x^2+1}$$

$$=\frac{1}{x}+4x+10+\frac{1}{4x+\dfrac{1}{x}}$$

Let $\dfrac{1}{x}+4x=t\Rightarrow t\ge 4\quad\therefore\ \Delta(x)|_{\min}=4+10+\dfrac{1}{4}=\dfrac{57}{4}$

17. (c) : 10 < x black, $10 - x$ red

$$\therefore\ \frac{{}^{x}C_1\cdot{}^{10-x}C_1}{{}^{10}C_2}=\frac{8}{15}\Rightarrow\frac{x(10-x)}{45}=\frac{8}{15}$$

$\Rightarrow\ x^2 - 10x + 24 = 0\ \Rightarrow x = 6$ or $x = 6$
Since given that number of black balls is more than red balls
$\therefore$ Number of BB = 6. Number of RB = 4

Now $P(E) = 1 - P(RR) = 1-\dfrac{{}^4C_2}{{}^{10}C_2}=\dfrac{39}{45}$

18. (c) : Let $L=\lim\limits_{n\to\infty}\dfrac{1}{m}\left(\dfrac{1}{n}.\dfrac{2}{n}.\dfrac{3}{n}...\dfrac{n}{n}\right)^{1/n}$

$$\Rightarrow\ \ln L=\lim_{n\to\infty}\left[\ln\left(\frac{1}{m}\right)+\frac{1}{n}\left(\ln\frac{1}{n}+\ln\frac{2}{n}+......+\ln\frac{n}{n}\right)\right]$$

$$=-\ln m+\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\ln\left(\frac{r}{n}\right)$$

$$=-\ln m+\int_0^1\ln x\,dx=-\ln m-1=\ln\left(\frac{1}{em}\right)\ \therefore L=\frac{1}{em}.$$

SOLUTIONS TO APRIL 2023 QUIZ CLUB

1. 0	**12.** ±4
2. π/4	**13.** π/4
3. 6	**14.** 8/3
4. 10	**15.** $f(x) + c$
5. 4^5	**16.** 3
6. $x = 3/2$	**17.** 1
7. 1	**18.** Pair of parallel planes
8. 47.5	**19.** 0.32
9. 36	**20.** $10\sqrt{3}$ cm^2/s.
10. 216	
11. π/2	

PRACTICE PAPER 2023

CUET (UG)

Exam on 21^{st} to 31^{st} May 2023

General Instructions

This practice paper contains two sections i.e. Section A and Section B [B1 and B2].
Section A has 15 questions covering both i.e. Mathematics/Applied Mathematics which is compulsory for all candidates.
Section B1 has 30 questions from Mathematics out of which 20 questions need to be attempted.
Section B2 has 30 questions purely from Applied Mathematics out of which 20 question need to be attempted.

SECTION A

1. A company manufactures two types of chemicals A and B. Each chemical requires the same type of raw materials P and Q. The table below shows number of units of P and Q required to manufacture one unit of A and one unit of B and the atmost availability of P and Q.

Chemical → / Raw material ↓	A	B	Availability
P	3	2	120
Q	2	5	160

The company gets profits of ₹ 350 and ₹ 400 by selling one unit of chemical A and one unit of chemical B respectively. If the entire production of A and B is sold, then formulate the problem as LPP.

(a) Maximize $z = 350x + 400y$ subject to
$3x + 2y \geq 120$, $2x + 5y \leq 160$, $x \geq 0$, $y \geq 0$
(b) Maximize $z = 350x + 400y$ subject to
$3x + 2y \leq 120$, $2x + 5y \leq 160$, $x \geq 0$, $y \geq 0$
(c) Maximize $z = 350x + 400y$ subject to
$3x + 2y \geq 120$, $2x + 5y \geq 160$, $x \geq 0$, $y \geq 0$
(d) Maximize $z = 350x + 400y$ subject to
$3x + 2y \leq 120$, $2x + 5y \geq 160$, $x \geq 0$, $y \geq 0$

2. If $y = x^2e^x$, then $\frac{d^2y}{dx^2}$ equals

(a) $-e^x(x^2 + 4x)$ (b) $e^{-x}(x^2 + 4x)$
(c) $e^x(x^2 + 4x + 2)$ (d) $e^x(2x + 4)$

3. A rod 108 metres long is bent to form a rectangle. Find its dimensions, if its area is maximum.

(a) 27 metres, 27 metres (b) 35 metres, 19 metres
(c) 31 metres, 23 metres (d) 25 metres, 21 metres

4. The value of $\int (e^{x \log a} + e^{a \log x} + e^{a \log a})\, dx$ is equal to

(a) $\frac{a^x}{\log a} + \frac{x^{a+1}}{a+1} + a^a x + c$

(b) $\frac{a^x}{\log a} + \frac{x^{a+1}}{a-1} + ax^a + c$

(c) $\frac{a^x}{\log a} + \frac{x^a}{a+1} + ax^a + c$

(d) $\frac{a^x}{\log x} + \frac{x^{a+1}}{a+1} + a^a x + c$

5. The probability distribution for a r.v. X is

$X = x$	0	1	2	3	4	5	6
$P(X = x)$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

Then $P(X \geq 2)$ equals

(a) $\frac{45}{49}$ (b) $\frac{15}{49}$ (c) $\frac{36}{49}$ (d) $\frac{1}{49}$

6. The solution of the differential equation $(x+1)\frac{dy}{dx} = 2xy$ is

(a) $\log y = x + \log |x| + c$
(b) $\log y = x + 2\log |x + 1| + c$
(c) $\log y = x - \log |x + 1| + c$
(d) $\log y = 2 [x - \log | x + 1| + c$

7. In a binomial distribution, if $n = 4$, $p = \frac{1}{2}$, then $P(X = 2)$ is

(a) $\frac{5}{8}$ (b) 1
(c) $\frac{3}{8}$ (d) None of these

8. The area bounded by the parabola $y^2 = 4x$ and the straight line $x + y = 3$ is

(a) $\frac{64}{3}$ sq. units (b) $\frac{8}{3}$ sq. units

(c) $\frac{14}{3}$ sq. units (d) $\frac{46}{3}$ sq. units

9. The standard deviation of the following probability distribution is

x_i	1	2	3	4
p_i	0.4	0.3	0.2	0.1

(a) 2 (b) 1 (c) 1.5 (d) 2.3

10. Which of the following statement(s) is/are correct?

(i) Adjoint of a symmetric matrix is symmetric,

(ii) Adjoint of a unit matrix is a unit matrix,

(iii) A(adj A) = (adj A) A = |A| I and

(iv) Adjoint of a diagonal matrix is a diagonal matrix.

(a) Only (i) (b) Only (ii)

(c) Only (iii) and (iv) (d) All of these

11. Consider the following statements :

Statement-1 : The area of the smaller region bounded by the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the line $\frac{x}{3}+\frac{y}{2}=1$ is $\frac{3}{2}(\pi-2)$ sq. units.

Statement-2 : Formula to calculate the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the line $\frac{x}{a}+\frac{y}{b}=1$ is $\frac{ab}{4}(\pi-2)$ sq. units.

In the light of above statements, choose the correct answer from the options given below :

(a) Both Statement I and Statement II are true.

(b) Both Statement I and Statement II are false.

(c) Statement I is true but Statement II is false.

(d) Statement I is false but Statement II is true.

12. Match the order and degree given in column II with corresponding differential equations given in column I and choose the correct option.

	Column I		Column II
(P)	$\{y'^3 + y''\}^2 = ay'$	(1)	Order = 2
(Q)	$3y = 7xy' + \frac{5}{y'}$	(2)	Order = 1
(R)	$y'' - 5y' + 6y = 0$	(3)	Degree = 2
(S)	$(y' - 4x)^{3/2} = x + 5y'$	(4)	Degree = 1
		(5)	Degree = 3

	P	Q	R	S
(a)	1,2,3	3,4	1,4	2,5
(b)	1,3	2,3	1,4	2,5
(c)	3,4	1,3	1,2	3,5
(d)	2,5	2,3	1,4	4,5

13. Match Column-I with Column-II and select the correct answer using the options given below.

	Column-I		Column-II
(P)	The maximum value of $f(x) = 9x(x-1)^2, 0 \le x \le 2$ is	(1)	–1
(Q)	The minimum value of $f(x) = \frac{x^2-1}{x^2+1}$ is	(2)	18
(R)	The local maximum value of $f(x) = \frac{x^2+1}{x^2-1}, x \in R-\{-1, 1\}$ is	(3)	–2
(S)	The local maximum value of $f(x) = \frac{x^2+1}{x}, x \in R-\{0\}$ is	(4)	2

	P	Q	R	S
(a)	1	2	2	4
(b)	2	1	1	3
(c)	2	1	1	4
(d)	1	2	3	4

14. Consider the following statements :

Statement-1 : If $y^2 = 3 + 2x - x^2$, then at (3, 0) and (–1, 0) tangent is perpendicular to x-axis.

Statement-2 : At (3, 0) and (–1, 0), $\frac{dy}{dx} = \infty$.

In the light of above statements, choose the correct answer from the options given below :

(a) Both Statement I and Statement II are true.

(b) Both Statement I and Statement II are false.

(c) Statement I is true but Statement II is false.

(d) Statement I is false but Statement II is true.

15. The curve $y = ax^3 + bx^2 + cx$ is inclined at 45° to x-axis at (0,0) but it touches x-axis at (1,0), then which of the following is correct statement?

(i) $f'(1) = 0$ (ii) $f''(1) = 2$

(iii) $f'''(2) = 12$ (iv) $f(2) = 2$

(a) Only (i) is correct

(b) Only (ii) and (iii) are correct

(c) only (i), (ii) and (iv) are correct

(d) All are correct

SECTION B1 (MATHEMATICS)

16. Let $f(x) = 2^{10}x + 1$ and $g(x) = 3^{10}x - 1$. If $fog(x) = x$, then x is equal to

(a) $\dfrac{3^{10}-1}{3^{10}-2^{-10}}$ (b) $\dfrac{2^{10}+1}{2^{10}-3^{-10}}$

(c) $\dfrac{1-3^{-10}}{2^{10}-3^{-10}}$ (d) $\dfrac{1-2^{-10}}{3^{10}-2^{-10}}$

17. The value of $\sin^{-1}[\cos(\sin^{-1}x)] + \cos^{-1}[\sin(\cos^{-1}x)]$ is

(a) 0 (b) $\pi/4$ (c) $\pi/2$ (d) π

18. If $A = \dfrac{1}{\pi}\begin{bmatrix} \sin^{-1}(\pi x) & \tan^{-1}\left(\dfrac{x}{\pi}\right) \\ \sin^{-1}\left(\dfrac{x}{\pi}\right) & \cot^{-1}(\pi x) \end{bmatrix}$

and $B = \dfrac{1}{\pi}\begin{bmatrix} -\cos^{-1}(\pi x) & \tan^{-1}\left(\dfrac{x}{\pi}\right) \\ \sin^{-1}\left(\dfrac{x}{\pi}\right) & -\tan^{-1}(\pi x) \end{bmatrix}$, then $A - B$ is equal to

(a) $\dfrac{1}{2}I$ (b) I (c) 0 (d) $2I$

19. Let $f(x) = \begin{cases} \dfrac{x^3+x^2-16x+20}{(x-2)^2}, & \text{if } x \neq 2 \\ b, & \text{if } x = 2 \end{cases}$

If $f(x)$ is continuous for all x, then b is equal to

(a) 7 (b) 3 (c) 2 (d) 5

20. Which of the following is a tangent to the curve given by $x^3 + y^3 = 2xy$?

(a) $y = x$ (b) $y = x + 2$

(c) $y = -x + 2$ (d) $y = -2x + 3/2$

21. If $\displaystyle\int \frac{f(x)}{\log\cos x}\,dx = -\log(\log\cos x) + C$,

then $f(x)$ is equal to

(a) $\tan x$ (b) $-\sin x$

(c) $-\cos x$ (d) $-\tan x$

22. For any vector $\vec{a}$, the value of

$(\vec{a}\times\hat{i})^2 + (\vec{a}\times\hat{j})^2 + (\vec{a}\times\hat{k})^2$ is

(a) $\vec{a}^2$ (b) $3\vec{a}^2$ (c) $4\vec{a}^2$ (d) $2\vec{a}^2$

23. If the lines $\dfrac{x+2}{4\lambda+1} = \dfrac{y-1}{4} = \dfrac{z}{-18}$ and $\dfrac{x}{-3} = \dfrac{y+1}{5\mu-3} = \dfrac{z-1}{6}$ are parallel to each other, then the value of the pair (λ, μ) is

(a) $\left(-2, \dfrac{1}{3}\right)$ (b) $\left(2, -\dfrac{1}{3}\right)$

(c) $\left(2, \dfrac{1}{3}\right)$ (d) cannot be found

24. Consider the linear programming problem
Max. $Z = 4x + y$
Subject to $x + y \le 50$; $x + y \ge 100$; $x, y \ge 0$
Then, the max value of Z is

(a) 0 (b) 50

(c) 100 (d) does not exist

25. If A and B are such that $P(A' \cup B') = \dfrac{2}{3}$ and $P(A \cup B) = \dfrac{5}{9}$, then $P(A') + P(B')$ is

(a) $\dfrac{5}{9}$ (b) $\dfrac{1}{9}$ (c) $\dfrac{10}{9}$ (d) $\dfrac{8}{9}$

26. Area of the region bounded by $y = |x|$ and $y = -|x| + 2$ is

(a) 4 sq. units (b) 3 sq. units

(c) 2 sq. units (d) 1 sq. unit

27. The differential equation of the family of circles with fixed radius 5 units and centre on the line $y = 2$ is

(a) $(x-2)^2 y'^2 = 25 - (y-2)^2$

(b) $(x-2)\, y'^2 = 25 - (y-2)^2$

(c) $(y-2)\, y'^2 = 25 - (y-2)^2$

(d) $(y-2)^2 y'^2 = 25 - (y-2)^2$

28. If $S = \left\{ x \in [0, 2\pi] : \begin{vmatrix} 0 & \cos x & -\sin x \\ \sin x & 0 & \cos x \\ \cos x & \sin x & 0 \end{vmatrix} = 0 \right\}$, then

$\displaystyle\sum_{x\in S} \tan\left(\frac{\pi}{3} + x\right)$ is equal to

(a) $-2+\sqrt{3}$ (b) $4+2\sqrt{3}$

(c) $-4-2\sqrt{3}$ (d) $-2-\sqrt{3}$

29. If $f : R \to R$, $g : R \to R$ and $h : R \to R$ are such that $f(x) = x^2$, $g(x) = \tan x$ and $h(x) = \log x$, then the value of $[go(foh)](x)$, if $x = 1$, will be

(a) 0 (b) 1 (c) -1 (d) π

30. For the function $f(x) = \begin{cases} \dfrac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}, & x \neq 0 \\ \cos x, & x = 0 \end{cases}$ consider the following statements:

Statement-I : $f(x)$ is discontinuous at $x = 0$.

Statement-II : $f(0) = 1$.

In the light of above statements, choose the correct answer from the options given below :

(a) Both Statement I and Statement II are true.
(b) Both Statement I and Statement II are false.
(c) Statement I is true but Statement II is false.
(d) Statement I is false but Statement II is true.

31. Consider the following statements:

Statement-I : $\int_{2016}^{2017} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{4033-x}}\,dx$ is equal to $\frac{1}{2}$.

Statement-II : $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

In the light of above statements, choose the correct answer from the options given below :

(a) Both Statement I and Statement II are true.
(b) Both Statement I and Statement II are false.
(c) Statement I is true but Statement II is false.
(d) Statement I is false but Statement II is true.

32. Match Column-I with Column-II and select the correct answer using the options given below.

Column-I		Column-II	
(P)	$\cos^{-1}\left(\cos\frac{46\pi}{7}\right) =$	(1)	$\frac{3\pi}{8}$
(Q)	$\tan^{-1}\left(\tan\left(-\frac{13\pi}{8}\right)\right) =$	(2)	$\frac{5\pi}{8}$
(R)	$\sin^{-1}\left(\sin\frac{33\pi}{7}\right) =$	(3)	$\frac{\pi}{6}$
(S)	$\cot^{-1}\left(\cot\left(\frac{-19\pi}{8}\right)\right) =$	(4)	$\frac{4\pi}{7}$
		(5)	$\frac{2\pi}{7}$

(a) P-5, Q-1, R-2, S-4
(b) P-4, Q-1, R-5, S-2
(c) P-1, Q-2, R-3, S-4
(d) P-2, Q-5, R-3, S-2

33. If $\begin{vmatrix} 1+x & x & x^2 \\ x & 1+x & x^2 \\ x^2 & x & 1+x \end{vmatrix} = px^5 + qx^4 + rx^3 + sx^2 + tx + w$,

then match the following columns.

Column-I		Column-II	
(P)	w is equal to	(1)	3
(Q)	t is equal to	(2)	1
(R)	$p + r + w$ is equal to	(3)	–2
(S)	$q + r$ is equal to	(4)	0

(a) P – 2, Q – 1, R – 4, S – 3
(b) P – 4, Q – 2, R – 3, S – 1
(c) P – 3, Q – 4, R – 1, S – 2
(d) P – 1, Q – 3, R – 2, S – 4

34. If A is square matrix, then which of the following statement(s) is/are correct?

(i) A + A' is symmetric
(ii) A + A' is skew symmetric
(iii) A - A' is symmetric
(iv) A - A' is skew symmetric

(a) Only (ii) is correct
(b) (i) and (iv) are correct
(c) (i), (iii) and (iv) are correct
(d) All are correct

35. If the relatiion R defined on the set N of natural numbers by $xRy \Leftrightarrow 2x^2 - 3xy + y^2 = 0$, then which of the following statement(s) is/are correct?

(i) R is symmetric
(ii) R is reflective
(iii) R is not symmetric
(iv) R is not reflective

(a) Only (ii) is correct
(b) Only (iii) is correct
(c) (ii) and (iii) are correct
(d) (ii) and (iv) are correct

Case Based MCQs

Case I : Read the following passage and answer the questions from 36 to 40.

Sonia wants to prepare a handmade gift box for her friend's birthday at home. For making lower part of box, she takes a square piece of cardboard of side 20 cm.

36. If x cm be the length of each side of the square cardboard which is to be cut off from corners of the square piece of side 20 cm, then possible value of x will be given by the interval

(a) [0, 20]
(b) (0, 10)
(c) (0, 3)
(d) None of these

37. Volume of the open box formed by folding up the cutting corner can be expressed as

(a) $V = x(20 - 2x)(20 - 2x)$
(b) $V = \frac{x}{2}(20 + x)(20 - x)$
(c) $V = \frac{x}{3}(20 - 2x)(20 + 2x)$
(d) $V = x(20 - 2x)(20 - x)$

38. The values of x for which $\frac{dV}{dx} = 0$, are

(a) 3, 4 (b) 0, $\frac{10}{3}$ (c) 0, 10 (d) 10, $\frac{10}{3}$

39. Sonia is interested in maximize the volume of the box. So, what should be the side of the square to be cut off so that the volume of the box is maximum?

(a) 12 cm (b) 8 cm (c) $\frac{10}{3}$ cm (d) 2 cm

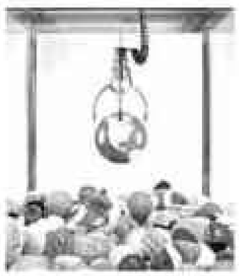
40. The maximum value of the volume is

(a) $\frac{17000}{27}$ cm^3 (b) $\frac{11000}{27}$ cm^3

(c) $\frac{8000}{27}$ cm^3 (d) $\frac{16000}{27}$ cm^3

Case II : Read the following passage and answer the questions from 41 to 45.

In a play zone, Alina is playing crane game. It has 12 blue soft toys, 8 red soft toys, 10 yellow soft toys and 5 green soft toys. Alina draws two soft toys one after the other without replacement.



41. What is the probability that the first soft toy is blue and the second soft toy is green?

(a) $\frac{5}{119}$ (b) $\frac{12}{119}$ (c) $\frac{6}{119}$ (d) $\frac{15}{119}$

42. What is the probability that the first soft toy is yellow and the second soft toy is red?

(a) $\frac{16}{119}$ (b) $\frac{8}{119}$

(c) $\frac{24}{119}$ (d) None of these

43. What is the probability that both the soft toys are red?

(a) $\frac{4}{85}$ (b) $\frac{24}{595}$ (c) $\frac{12}{119}$ (d) $\frac{64}{119}$

44. What is the probability that the first soft toy is green and the second soft toy is not yellow?

(a) $\frac{10}{119}$ (b) $\frac{6}{85}$

(c) $\frac{12}{119}$ (d) None of these

45. What is the probability that both the soft toys are not blue?

(a) $\frac{6}{595}$ (b) $\frac{12}{85}$ (c) $\frac{15}{17}$ (d) $\frac{253}{595}$

SECTION B2 (APPLIED MATHEMATICS)

16. A is a matrix of the type 4×6 and R is a row of A, then what is the type of R as a matrix?

(a) Column matrix, 4×1
(b) Row matrix, 1×4
(c) Column matrix, 1×6
(d) Row matrix, 1×6

17. Find the average speed of a boat in a round trip between two places 22 km apart if the speed of boat in still water is 11 km/h and speed of the stream is 6 km/h.

(a) 11 km/h (b) 7.73 km/h
(c) 6.75 km/h (d) 8.5 km/h

18. Find the last digit of 12^{12}.

(a) 6 (b) 8 (c) 4 (d) 2

19. If $x^4 + y^4 = 16$, then find the second order derivative of y.

(a) $y'' = \frac{-24x^2}{y^7}$ (b) $y'' = \frac{-16x^3}{y^5}$

(c) $y'' = \frac{-48x^2}{y^7}$ (d) $y'' = \frac{-48x^3}{y^5}$

20. Let X denote the number of hours Ramesh study on a particular day. Also it is known that

$$P(X = x) = \begin{cases} 0.1, & \text{if } x = 0 \\ k(x+1), & \text{if } x = 1 \text{ or } 2 \\ k(6-x), & \text{if } x = 3 \text{ or } 4 \\ 0 & \text{Otherwise} \end{cases},$$

where k is constant.

Consider the following statements.

I. The value of k is 0.09.
II. The probability that Ramesh study atleast two hours is 0.27.
III. The probability that Ramesh study atmost two hours is 0.55.
IV. The probability that Ramesh study exactly two hours is 0.72.

Which of the given statement(s) is/are correct?

(a) only I (b) only I and II
(c) only I and III (d) All are correct

21. Which of the following statement(s) is/are correct about average prices if the price index is 118?

(a) The prices have increased by 118%.
(b) The prices have decreased by 18%.
(c) The prices have increased by 1.8%.
(d) The prices have increased by 18%.

22. A washing machine costing ₹ 45000 has a useful life of 15 years. If its scrap value is ₹ 3000, then find the annual depreciation.

(a) ₹ 2800 (b) ₹ 3000
(c) ₹ 3200 (d) None of these

23. The corner points of the feasible region determined by the following systems of linear inequalities :
$2x + y \leq 10$, $x + 3y \leq 15$, $x \geq 0$, $y \geq 0$ are (0, 0), (5, 0), (3, 4) and (0, 5).
Let $Z = px + qy$, when $p, q > 0$, then find the relation between p and q so that the maximum of Z occurs at both points (3, 4) and (0, 5).

(a) $3q = p$ (b) $p = q$
(c) $3p = q$ (d) $2p = 3q$

24. Consider the following statements:
Statement I : Fisher's ideal index number satisfied both time reversal and factor reversal test.
Statement II : The time reversal test is satisfied if $P_{01} \times P_{10} \neq 1$.
In the light of above statements, choose the correct answer from the options given below.

(a) Both Statement I and II are true
(b) Both Statement I and II are false.
(c) Statement I is true but Statement II is false.
(d) Statement I is false but Statement II is true.

25. A, B and C started a business. A invested ₹ 650000 for 6 months, B invested ₹ 840000 for 5 months and C invested ₹ 1000000 for 3 months. A is a working partner and he received 5% of the profit as incentive. The profit earned was ₹ 74000. Calculate B's share in the profit.

(a) ₹ 24500 (b) ₹ 26600
(c) ₹ 27780 (d) ₹ 27890

26. In the given two columns, column-II represents critical points of the function given in column-I. Match the following columns by choosing an appropriate option.

Column-I		Column-II	
(P)	$x^3 - 2x^2 - 4x - 1$	(1)	$x = 0$
(Q)	$5x^2 - 3x + 1$	(2)	$x = 1, -1$
(R)	$x^2 - 2$	(3)	$x = 2, \frac{-2}{3}$
(S)	$x + \frac{1}{x}$	(4)	$x = \frac{3}{10}$

(a) (P) - (2), (Q) - (4), (R) - (1), (S) - (3)
(b) (P) - (3), (Q) - (4), (R) - (4), (S) - (2)
(c) (P) - (3), (Q) - (1), (R) - (4), (S) - (2)
(d) (P) - (2), (Q) - (1), (R) - (3), (S) - (4)

27. What sum of money is needed to invest now, so as to get ₹ 7000 at the beginning of every quarter forever, if the money is worth 5.6% per annum compound quarterly?

(a) ₹ 507000 (b) ₹ 1507000
(c) ₹ 5007000 (d) None of these

28. The value of (x_1, x_2) for an optimal solution for minimize $Z = 5x + 10y$ subject to the constraints $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x \geq 0$, $y \geq 0$ is

(a) (60, 30) (b) (40, 20)
(c) (60, 0) (d) (120, 0)

29. Consider the following statements:
Statement I : A sample from the population does not have to share the same characteristics as the population.
Statement II :A method of using sample to estimate population parameters is known as statistical inference.
In the light of above statements, choose the correct answer from the options given below

(a) Both Statement I and II are true
(b) Both Statement I and II are false.
(c) Statement I is true but Statement II is false.
(d) Statement I is false but Statement II is true.

30. A man wants to cut three lengths from a single piece of cardboard of length 87 cm. The second length is to be 3 cm longer than the shortest and the third length is to be twic as long as the shortest. What are the least possible lengths for the shortest board if the third piece is to be atleast 5 cm longer than the second?

(a) 8 cm (b) 21 cm (c) 9 cm (d) 20 cm

31. The cost of manufacturing x units of a commodity is $48 + 15x + 3x^2$. Find the output where average cost and marginal cost are equal.

(a) 7 (b) 6 (c) 5 (d) 4

32. Two number are selected at random (without replacement) from the first 5 positive integers. Let X denote the larger of the two numbers obtained. Find the mean of this distribution.

(a) 1 (b) 2 (c) 4 (d) 8

33. A runs 3 times as fast as *B*. If *A* gives *B* a start of 50 m. Find how far must be the finish point on the race course so that *A* and *B* reach the goal at the same time?
(a) 100 m (b) 75 m (c) 65 m (d) 56 m

34. A bond of face value ₹2000 has a coupon rate of 8% per annum with interest paid semi-annually and matures in 5 years. If the bond is priced to yield 10% p.a., then find the value of bond.
(Given $(1.05)^{-10} = 0.6139$)
(a) ₹ 1845.56 (b) ₹ 1865.23
(c) ₹ 1910.52 (d) None of these

35. Match the following Columns and choose the correct option.

Column-I		Column -II	
(P)	Singular matrix	(1)	Determinant is equal to one
(Q)	Identity matrix	(2)	Determinant is not defined
(R)	Non square matrix	(3)	Determinant is equal to zero
(S)	Upper triangular matrix	(4)	Determinant is equal to the product of diagonal entries

(a) (P) - (3), (Q) - (4), (R) - (2), (S) - (1)
(b) (P) - (1), (Q) - (3), (R) - (2), (S) - (4)
(c) (P) - (2), (Q) - (1), (R) - (4), (S) - (1)
(d) (P) - (3), (Q) - (1), (R) - (2), (S) - (4)

36. Three pipes *A*, *B* and *C* are installed to fill a tank. Pipes *A* and *B* opened together can fill the tank in the same time in which *C* can alone fill the tank. If pipe *B* can fill the tank 15 minutes faster than pipe *A* and 5 minutes slower than pipe *C*, then find the time required by pipe *A* to fill the tank alone.
(a) 40 min. (b) 30 min. (c) 27 min. (d) 25 min.

37. Find the derivative of $4x^3$ w.r.t x^4.
(a) 1 (b) x (c) $\frac{3}{x}$ (d) $\frac{2}{3x^2}$

38. Mr. Dharmendra Patel purchase a motorcycle of ₹ 150000 with ₹ 25000 down payment. He wishes to repay balance in equal monthly payments in 4 years. If bank charges 9% p.a. compounded monthly, calculate the EMI. (Given $(1.0075)^{48} = 1.4314$)
(a) ₹ 3110.66 (b) ₹ 4206.21
(c) ₹ 3526.76 (d) ₹ 4102.12

39. Consider the following statements.
I. Saving accounts opens for necessarily a specific purpose.
II. Sinkings funds are set up for specific upcoming expenses.
III. Sinking funds are meant for large unexpected expenses.
IV. Savings accounts are set up for long-term savings
Which of the given statements(s) is/are true?
(a) IV only (b) I, II and IV
(c) II and IV (d) II, III, IV

40. Evaluate (29 + 42) mod 11.
(a) 16 (b) 5
(c) 6 (d) 11

41. Calculate the difference between first four years and first three years moving average for the following series of observations.

Year	2015	2016	2017	2018	2019
Demand	55	62	54	69	72

(a) 4.66 (b) 4.25
(c) 3.76 (d) 3

42. In what ratio must a grocer mix two varieties of sugar worth ₹ 50 per kg and ₹ 60 per kg respectively so that by the selling the mixture at ₹ 57.20 per kg, he may gain 10%?
(a) 3 : 2 (b) 3 : 1
(c) 5 : 2 (d) 4 : 1

43. Find the integral value of *x* if

$$\begin{bmatrix} x & 4 & -1 \end{bmatrix} \begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 0 \\ 2 & 2 & 4 \end{bmatrix} \begin{bmatrix} x & 4 & -1 \end{bmatrix}^T = O.$$

(a) 4 (b) −4
(c) $\frac{1}{2}, -4$ (d) $\frac{1}{2}, 4$

44. Solve the inequality : $-8 \le 5x - 3 < 7$
(a) (−1, 2) (b) (−2, −1]
(c) [−1, 2) (d) None of these

45. Increase in the number of patients in the hospital due to heat stroke is
(a) Secular trend (b) Cyclical variation
(c) Seasonal variation (d) Irregular variation

SOLUTIONS

SECTION A

1. (b): Let x units of chemical A and y units of chemical B are being manufactured to maximize the profit.
The total profit z on x units of A and y units of B is

$z = 350x + 400y$

Hence, the LPP is formulated as
Maximize, $z = 350x + 400y$
Subject to, $3x + 2y \leq 120$, $2x + 5y \leq 160$, $x \geq 0$, $y \geq 0$

2. (c) : $y = x^2 e^x$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(x^2 e^x) = x^2 \frac{d}{dx}(e^x) + e^x \frac{d}{dx}(x^2)$$

$$= x^2 \cdot e^x + e^x \times 2x = e^x(x^2 + 2x)$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}[e^x(x^2 + 2x)]$$

$$= e^x \cdot \frac{d}{dx}(x^2 + 2x) + (x^2 + 2x) \cdot \frac{d}{dx}(e^x)$$

$= e^x \cdot (2x + 2) + (x^2 + 2x) \cdot e^x$
$= e^x(2x + 2 + x^2 + 2x) = e^x(x^2 + 4x + 2)$

3. (a) : Let x be the length and y be the breadth of the rectangle

$\therefore \quad 2x + 2y = 108 \Rightarrow y = 54 - x$

Now, area of rectangle $= xy = x(54 - x)$
Let $f(x) = 54x - x^2$
$\therefore \quad f'(x) = 54 - 2x$ and $f''(x) = -2$
For critical point $f'(x) = 0$
$\Rightarrow \quad 54 - 2x = 0 \Rightarrow x = 27$
$f''(27) = -2 < 0$
Hence, length and breadth both are equal to 27 m.

4. (a) : Let $I = \int (e^{x \log a} + e^{a \log x} + e^{a \log a})\, dx$

$$= \int (e^{\log a^x} + e^{\log x^a} + e^{\log a^a})\, dx$$

$$= \int (a^x + x^a + a^a)\, dx \qquad [\because e^{\log \lambda} = \lambda]$$

$$= \int a^x dx + \int x^a dx + \int a^a dx = \frac{a^x}{\log a} + \frac{x^{a+1}}{a+1} + a^a x + c$$

5. (a) : $\because \sum_{x=0}^{6} P(X = x) = 1$

$\Rightarrow \quad P(X = 0) + P(X = 1) + ... + P(X = 6) = 1$
$\Rightarrow \quad k + 3k + 5k + 7k + 9k + 11k + 13k = 1$

$$\Rightarrow \quad 49k = 1 \;\Rightarrow\; k = \frac{1}{49}$$

$P(X \geq 2) = P(X = 2) + P(X = 3) \; .... + P(X = 6)$

$$= 5k + 7k + 9k + 11k + 13k = 45k = \frac{45}{49}$$

6. (d): We have, $(x+1)\frac{dy}{dx} = 2xy$

$$\Rightarrow \quad (x + 1)dy = 2xy\, dx \Rightarrow \frac{dy}{y} = \frac{2x}{x+1}dx, \text{ if } x \neq -1$$

$$\Rightarrow \quad \int \frac{1}{y} dy = 2\int \frac{x}{x+1} dx \qquad \text{[Integrating both sides]}$$

$$\Rightarrow \quad \int \frac{1}{y} dy = 2\int \left(1 - \frac{1}{x+1}\right)dx$$

$\Rightarrow \quad \log y = 2[x - \log |x + 1|] + c$
Clearly, it is defined for all $x \in R - \{-1\}$.
Hence, $\log y = 2[x - \log|x + 1|] + c$, is the solution of the given differential equation.

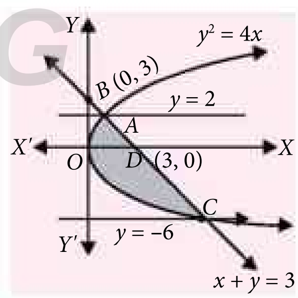
7. (c) : Here $n = 4$ and $p = \frac{1}{2}$

So, $q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

$$\therefore \quad P(X = 2) = {}^4C_2\, p^2 q^2 = 6 \times \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^2$$

$$= 6 \times \frac{1}{4} \times \frac{1}{4} = \frac{3}{8}$$

8. (a) : Equation of parabola is $y^2 = 4x$ ...(i)
Equation of line is $x + y = 3$ ...(ii)



Curve (i) is a right handed parabola whose vertex is (0, 0) and axis is $y = 0$.
Line (ii) cuts x-axis at (3, 0) and y-axis at (0, 3).
Here required area $OCDAO$ is bounded by curve (i) and (ii) Putting the value of x from equation (ii) in (i), we get
$y^2 = 4(3 - y)$
$\Rightarrow \quad y^2 + 4y - 12 = 0 \;\Rightarrow y = -6, 2$
Required area $OCDAO$

$$= \int_{-6}^{2} \left[(3 - y) - \frac{y^2}{4}\right] dy = \left[3y - \frac{y^2}{2} - \frac{y^3}{12}\right]_{-6}^{2}$$

$$= \left(6 - 2 - \frac{2}{3}\right) - \left(-18 - 18 + \frac{216}{12}\right) = \frac{64}{3} \text{ sq. units}$$

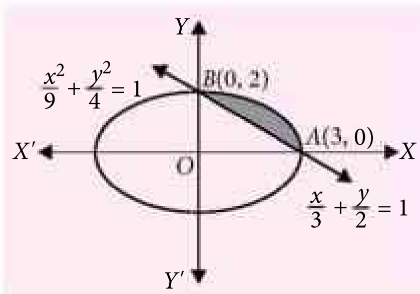
9. (b): Mean, $\mu = \Sigma p_i x_i$
$= (0.4 \times 1) + (0.3 \times 2) + (0.2 \times 3) + (0.1 \times 4) = 2$
Variance, $\sigma^2 = \Sigma p_i x_i^2 - (\Sigma p_i x_i)^2$
$= 0.4\,(1) + 0.3\,(4) + 0.2(9) + 0.1\,(16) - 4 = 5 - 4 = 1$
Standard deviation, $\sigma = \sqrt{1} = 1$

10. (d) : All are correct statements.

11. (a) : Given curve is $\frac{x^2}{9}+\frac{y^2}{4}=1$, represents an ellipse with centre at (0, 0).

and equation of line is $\frac{x}{3}+\frac{y}{2}=1$

Thus, intersection points are $A(3, 0)$ and $B(0, 2)$.



$\therefore$ Required area = area of shaded region

$$=\int_0^3 2\sqrt{1-\frac{x^2}{9}}\,dx-\int_0^3 2\left(1-\frac{x}{3}\right)dx$$

$$=\frac{2}{3}\int_0^3\sqrt{3^2-x^2}\,dx-\frac{2}{3}\int_0^3(3-x)\,dx$$

$$=\frac{2}{3}\left[\frac{x}{2}\sqrt{3^2-x^2}+\frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3-\frac{2}{3}\left[3x-\frac{x^2}{2}\right]_0^3$$

$$=\frac{2}{3}\left[0+\frac{9}{2}\sin^{-1}(1)-0\right]-\frac{2}{3}\left[9-\frac{9}{2}-0\right]$$

$$=3\left(\frac{\pi}{2}\right)-(3)=3\left(\frac{\pi}{2}-1\right)=\frac{3}{2}(\pi-2)\text{ sq. units}$$

12. (b) : (P) $\{(y')^3+y''\}^2=ay'$

$\Rightarrow$ $(y')^6+(y'')^2+2(y')^3y''=ay'$

$\Rightarrow$ Order is '2' and degree is '2'.

(Q) $3y=7xy'+\frac{5}{y'}$

$\Rightarrow$ $3yy'=7x(y')^2+5$

$\Rightarrow$ Order is '1' and degree is '2'.

(R) $y''-5y'+6y=0$

$\Rightarrow$ Order is '2' and degree is '1'

(S) $(y'-4x)^{3/2}=x+5y'$

$\Rightarrow$ $(y')^3-(4x)^3-3\times 4x(y')(y'-4x)$

$=x^2+25(y')^2+10xy'$

$\Rightarrow$ Order is '1' and degree is '3'.

13. (b) : (P) $f'(x)=9(3x^2-4x+1)=0\Rightarrow x=1,\frac{1}{3}$

$f(1)=0, f(0)=0, f(2)=18\Rightarrow f_{max}=18$

$\therefore$ $f\left(\frac{1}{3}\right)=9\cdot\frac{1}{3}\cdot\frac{4}{9}=\frac{4}{3}$

$f(1)=0, f(0)=0, f(2)=18$

$\therefore$ $f_{max}=18$

(Q) $f'(x)=\frac{4x}{(x^2+1)^2}=0\Rightarrow x=0$

$\therefore$ $f_{min}=f(0)=-1$

(R) $f'(x)=-\frac{4x}{(x^2-1)^2}=0\Rightarrow x=0$

$\therefore$ $f_{max}=f(0)=-1$

(S) $f'(x)=1-\frac{1}{x^2}=0\Rightarrow x=\pm 1$

$\therefore$ $f_{max}=f(-1)=-2$ and $f_{min}=f(1)=2$

14. (a) : Here $y^2=3+2x-x^2$

$\Rightarrow$ $2y\frac{dy}{dx}=2-2x\Rightarrow\frac{dy}{dx}=\left(\frac{1-x}{y}\right)$

$\left(\frac{dy}{dx}\right)_{(3,0)}=\infty$ and $\left(\frac{dy}{dx}\right)_{(-1,0)}=\infty$

i.e., Tangents makes an angle $\frac{\pi}{2}$ with x-axis.

15. (c) : $\frac{dy}{dx}=3ax^2+2bx+c\Rightarrow\left.\frac{dy}{dx}\right|_{(\text{at }x=0)}=\tan 45°=1$

$\therefore$ $c=1$

Now, $\left[\frac{dy}{dx}\right]_{(1,0)}=3a+2b+c=0$ as x-axis is tangent.

$\therefore$ $3a+2b+1=0$ $\quad[\because c=1]$ ...(i)

Now, (1, 0) lies on curve,

$\therefore$ $a+b+1=0$ ...(ii)

Solving (i) and (ii), we get $a=1$, $b=-2$

$\therefore$ $f(x)=x^3-2x^2+x$ and $f'(x)=3x^2-4x+1$

Also, $f''(x)=6x-4$ and $f'''(x)=6$

Now, $f(2)=2^3-2(2)^2+2=8-8+2=2$

$f'(1)=3(1)^2-4(1)+1=3-4+1=0$

$f''(1)=6(1)-4=6-4=2$ and $f''(2)=6$

SECTION B1 (MATHEMATICS)

16. (d) : We have, $f(g(x))=x$

$\Rightarrow$ $f(3^{10}x-1)=x\Rightarrow 2^{10}(3^{10}x-1)+1=x$

$\Rightarrow$ $x(2^{10}3^{10}-1)=2^{10}-1$

$\Rightarrow$ $x=\frac{2^{10}-1}{2^{10}3^{10}-1}\Rightarrow x=\frac{2^{10}(1-2^{-10})}{2^{10}(3^{10}-2^{-10})}$

$\Rightarrow$ $x=\frac{1-2^{-10}}{3^{10}-2^{-10}}$

17. (c) : Consider,

$\sin^{-1}[\cos(\sin^{-1}x)] + \cos^{-1}[\sin(\cos^{-1}x)]$

$$= \sin^{-1}\left[\sin\left(\frac{\pi}{2}-\sin^{-1}x\right)\right]+\cos^{-1}\left[\cos\left(\frac{\pi}{2}-\cos^{-1}x\right)\right]$$

$$= \frac{\pi}{2} - \sin^{-1}x + \frac{\pi}{2} - \cos^{-1}x = \pi - (\sin^{-1}x + \cos^{-1}x)$$

$$= \pi - \frac{\pi}{2} = \frac{\pi}{2} \qquad \left[\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$$

18. (a) : $A - B$

$$= \frac{1}{\pi}\begin{bmatrix} \sin^{-1}(\pi x)+\cos^{-1}(\pi x) & 0 \\ 0 & \cot^{-1}(\pi x)+\tan^{-1}(\pi x) \end{bmatrix}$$

$$= \frac{1}{\pi}\begin{bmatrix} \frac{\pi}{2} & 0 \\ 0 & \frac{\pi}{2} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \frac{1}{2}I$$

19. (a) : $f(x) = \begin{cases} \dfrac{x^3+x^2-16x+20}{(x-2)^2}, & \text{if } x \neq 2 \\ b, & \text{if } x = 2 \end{cases}$

$$\therefore \lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{x^3+x^2-16x+20}{(x-2)^2}$$

$$= \lim_{x\to 2} \frac{(x-2)(x^2+3x-10)}{(x-2)^2} = \lim_{x\to 2} \frac{(x-2)(x+5)(x-2)}{(x-2)^2}$$

$$= \lim_{x\to 2}(x+5) = 2+5 = 7$$

$\because$ $f(x)$ is continuous for all x.

$\therefore$ $f(2) = \lim_{x\to 2} f(x) \Rightarrow b = 7$

20. (c) : Given curve is $x^3 + y^3 = 2xy$ ...(i)

Differentiating (i) with respect to 'x', we get

$$3x^2 + 3y^2\frac{dy}{dx} = 2x\frac{dy}{dx} + 2y \Rightarrow \frac{dy}{dx}(3y^2 - 2x) = 2y - 3x^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{2y-3x^2}{3y^2-2x}$$

Now, solving (i) with $y = -x + 2$, we get $x = 1$ and $y = 1$

$$\therefore \left(\frac{dy}{dx}\right)_{(1,1)} = \frac{2\times 1 - 3\times 1^2}{3\times 1^2 - 2\times 1} = \frac{2-3}{3-2} = -1,$$

which is equal to slope of the line $y = -x + 2$.

So, line $y = -x + 2$ is tangent to $x^3 + y^3 = 2xy$.

21. (a) : We have, $\int \frac{f(x)}{\log\cos x}dx = -\log(\log\cos x) + C$

Differentiating both sides w.r.t. x, we get

$$\frac{f(x)}{\log\cos x} = \frac{-1}{\log\cos x}\times\frac{1}{\cos x}\times(-\sin x)$$

$$\Rightarrow \frac{f(x)}{\log\cos x} = \frac{\tan x}{\log\cos x} \Rightarrow f(x) = \tan x$$

22. (d) : Consider, $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$

$\therefore$ $\vec{a}^2 = x^2 + y^2 + z^2$ ...(i)

$$\text{Now, } \vec{a}\times\hat{i} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 1 & 0 & 0 \end{vmatrix}$$

$$= \hat{i}(0) - \hat{j}(-z) + \hat{k}(-y) = z\hat{j} - y\hat{k}$$

$\therefore$ $(\vec{a}\times\hat{i})^2 = y^2 + z^2$

Similarly, $(\vec{a}\times\hat{j})^2 = x^2 + z^2$

and $(\vec{a}\times\hat{k})^2 = x^2 + y^2$

$\therefore$ $(\vec{a}\times\hat{i})^2 + (\vec{a}\times\hat{j})^2 + (\vec{a}\times\hat{k})^2$

$= y^2 + z^2 + x^2 + z^2 + x^2 + y^2 = 2(x^2 + y^2 + z^2) = 2\vec{a}^2$ [Using (i)]

23. (c) : Consider, $L_1 : \frac{x+2}{4\lambda+1} = \frac{y-1}{4} = \frac{z}{-18}$

and $L_2 : \frac{x}{-3} = \frac{y+1}{5\mu-3} = \frac{z-1}{6}$

If two lines are parallel, then their direction ratios are proportional.

$$\therefore \frac{4\lambda+1}{-3} = \frac{4}{5\mu-3} = \frac{-18}{6}$$

$$\Rightarrow \frac{4\lambda+1}{-3} = -3 \text{ and } \frac{4}{5\mu-3} = -3$$

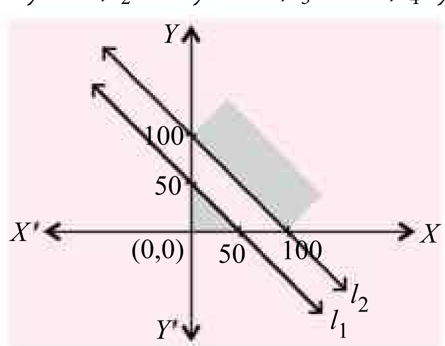
$\Rightarrow$ $4\lambda + 1 = 9$ and $4 = -15\mu + 9$

$\Rightarrow$ $4\lambda = 8$ and $15\mu = 5$ $\Rightarrow$ $\lambda = 2$ and $\mu = \frac{1}{3}$

So, the value of the pair (λ, μ) is $\left(2, \frac{1}{3}\right)$

24. (d) : Convert the given inequation into equation.

Let $l_1 : x + y = 50$; $l_2 : x + y = 100$; $l_3 : x = 0$; $l_4 : y = 0$



Since, no feasible region determined, hence no maximum value of Z exists.

25. (c) : We have, $P(A' \cup B') = \frac{2}{3}$ and $P(A \cup B) = \frac{5}{9}$

Since, $P(A' \cup B') = P(A \cap B)' = 1 - P(A \cap B)$

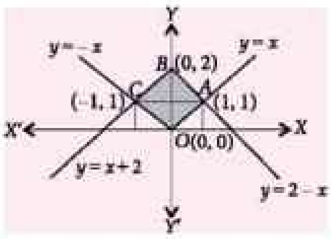
$\Rightarrow \frac{2}{3} = 1 - P(A \cap B) \Rightarrow P(A \cap B) = 1 - \frac{2}{3} = \frac{1}{3}$

$\because P(A') + P(B') = 1 - P(A) + 1 - P(B)$

$= 2 - [P(A) + P(B)] = 2 - [P(A \cup B) + P(A \cap B)]$

$= 2 - \left(\frac{5}{9} + \frac{1}{3}\right) = 2 - \left(\frac{5+3}{9}\right) = \frac{18-8}{9} = \frac{10}{9}$

26. (c) : Here, $OABCO$ is the bounded region.
Required area = area ($OABCO$) = 2 × Area ($OABO$)



$= 2\left[\int_0^1 y\, dy + \int_1^2 (2-y) dy\right]$

$= 2\left[\frac{y^2}{2}\right]_0^1 + 2\left[2y - \frac{y^2}{2}\right]_1^2$

$= [1 - 0] + 2\left[4 - 2 - 2 + \frac{1}{2}\right]$

= 1 + 1 = 2 sq. units

27. (d) : The equation of circle is $(x - \alpha)^2 + (y - 2)^2 = 25$ ...(i)

Differentiating w.r.t. x, we get

$(x-\alpha) + (y-2)\frac{dy}{dx} = 0 \Rightarrow x - \alpha = -(y-2)\frac{dy}{dx}$ ...(ii)

From (i) and (ii) on eliminating 'α'

$(y-2)^2\left(\frac{dy}{dx}\right)^2 + (y-2)^2 = 25$

$\Rightarrow (y-2)^2(y')^2 = 25 - (y-2)^2$

28. (d) : Let $A = \begin{bmatrix} 0 & \cos x & -\sin x \\ \sin x & 0 & \cos x \\ \cos x & \sin x & 0 \end{bmatrix}$

$\because |A| = 0 \Rightarrow 0(0 - \sin x \cos x) - \cos x(0 - \cos^2 x) - \sin x(\sin^2 x - 0) = 0$

$\Rightarrow \cos^3 x - \sin^3 x = 0 \Rightarrow \tan^3 x = 1 \Rightarrow \tan x = 1$

$\sum_{x \in s} \frac{\sqrt{3} + \tan x}{1 - \sqrt{3}\tan x} = \frac{\sqrt{3}+1}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}} = \frac{1+3+2\sqrt{3}}{1-3}$

$= -\frac{4}{2} - \frac{2\sqrt{3}}{2} = -2 - \sqrt{3}$

29. (a) : $[go(foh)](x) = gof(h(x)) = gof(\log x)$

$= g(\log x)^2 = \tan(\log x)^2 = \tan(\log 1)^2 = 0$

30. (a) : Statement-II is true as $\cos 0 = 1$.
Now, consider statement -I

$$\lim_{x \to 0^+} \frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1} = \lim_{h \to 0} \frac{e^{\frac{1}{h}} - 1}{e^{\frac{1}{h}} + 1} = \lim_{h \to 0} \frac{1 - e^{\frac{-1}{h}}}{1 + e^{\frac{-1}{h}}} = 1.$$

And $\lim_{x \to 0^-} \frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1} = \lim_{h \to 0} \frac{e^{\frac{-1}{h}} - 1}{e^{\frac{-1}{h}} + 1} = -1$

Thus, L.H.L. ≠ R.H.L. The function has non-removable discontinuity at $x = 0$.

31. (a) : Clearly, Statement-II is true.

Now, let $I = \int_{2016}^{2017} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{4033 - x}} dx$ ...(i)

$\Rightarrow I = \int_{2016}^{2017} \frac{\sqrt{4033 - x}}{\sqrt{4033 - x} + \sqrt{x}} dx$ ...(ii)

$\left[\text{Using, } \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right]$

Adding (i) and (ii), we get

$2I = \int_{2016}^{2017} \frac{\sqrt{x} + \sqrt{4033 - x}}{\sqrt{x} + \sqrt{4033 - x}} dx \Rightarrow 2I = \int_{2016}^{2017} 1\, dx$

$\Rightarrow 2I = [x]_{2016}^{2017} = 2017 - 2016 \Rightarrow I = \frac{1}{2}$

32. (b) : (P) $\cos^{-1}\left(\cos\frac{46\pi}{7}\right) = \cos^{-1}\left(\cos\left(6\pi + \frac{4\pi}{7}\right)\right)$

$= \cos^{-1}\left(\cos\frac{4\pi}{7}\right) = \frac{4\pi}{7}$

(Q) $\tan^{-1}\left(\tan\left(\frac{-13\pi}{8}\right)\right) = \tan^{-1}\left(-\tan\left(2\pi - \frac{3\pi}{8}\right)\right)$

$= \tan^{-1}\left(\tan\left(\frac{3\pi}{8}\right)\right) = \frac{3\pi}{8}$

(R) $\sin^{-1}\left(\sin\frac{33\pi}{7}\right) = \sin^{-1}\left(\sin\left(4\pi + \frac{5\pi}{7}\right)\right)$

$= \sin^{-1}\left(\sin\frac{5\pi}{7}\right)$

$= \sin^{-1}\left(\sin\left(\pi - \frac{2\pi}{7}\right)\right) = \sin^{-1}\left(\sin\frac{2\pi}{7}\right) = \frac{2\pi}{7}$

(S) $\cot^{-1}\left(\cot\left(\frac{-19\pi}{8}\right)\right) = \cot^{-1}\left(-\cot\left(2\pi + \frac{3\pi}{8}\right)\right)$

$$= \cot^{-1}\left(-\cot\frac{3\pi}{8}\right) = \cot^{-1}\left(-\cot\left(\pi - \frac{5\pi}{8}\right)\right)$$

$$= \cot^{-1}\left(\cot\frac{5\pi}{8}\right) = \frac{5\pi}{8}$$

33. (a) : We have, $\begin{vmatrix} 1+x & x & x^2 \\ x & 1+x & x^2 \\ x^2 & x & 1+x \end{vmatrix}$

$= px^5 + qx^4 + rx^3 + sx^2 + tx + w$

Consider, $\begin{vmatrix} 1+x & x & x^2 \\ x & 1+x & x^2 \\ x^2 & x & 1+x \end{vmatrix}$

$$= \begin{vmatrix} 1+2x+x^2 & x & x^2 \\ 1+2x+x^2 & 1+x & x^2 \\ 1+2x+x^2 & x & 1+x \end{vmatrix} \text{ [Applying } C_1 \to C_1 + C_2 + C_3]$$

$$= (1+2x+x^2)\begin{vmatrix} 1 & x & x^2 \\ 1 & 1+x & x^2 \\ 1 & x & 1+x \end{vmatrix}$$

[Taking out $(1 + 2x + x^2)$ common from C_1]

$$= (1+2x+x^2)\begin{vmatrix} 0 & -1 & 0 \\ 0 & 1 & x^2-1-x \\ 1 & x & 1+x \end{vmatrix}$$

[$\because R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$]

$= (1 + 2x + x^2)(1 + x - x^2)$

$= 1 + x - x^2 + 2x + 2x^2 - 2x^3 + x^2 + x^3 - x^4$

$= 0 \cdot x^5 - 1 \cdot x^4 - 1 \cdot x^3 + 2 \cdot x^2 + 3x + 1$

Comparing with $px^5 + qx^4 + rx^3 + sx^2 + tx + w$, we get

$p = 0, q = -1, r = -1, s = 2, t = 3, w = 1$

(P) $w = 1$ **(Q)** $t = 3$

(R) $p + r + w = 0 - 1 + 1 = 0$

(S) $q + r = -1 - 1 = -2$

34. (b) : Let $P = A + A'$

Then, $P' = (A + A')' = A' + (A')'$ [$\because (A + B)' = A' + B'$]

$= A' + A$ [...$(A')' = A$]

$= A + A' = P$. $\therefore$ P is symmetric.

Also, let $Q = A - A'$.

Then, $Q' = (A - A')' = A' - (A')' = A' - A = -(A - A') = -Q$.

$\therefore$ Q is skew-symmetric.

35. (c) : (i) $xRx \Leftrightarrow 2x^2 - 3xx + x^2 = 0 \ \forall \ x \in N$.

$\therefore$ R is reflexive

(ii) For $x = 1, y = 2$; $1R2 \Leftrightarrow 2 \cdot 1^2 - 3(1)(2) + 2^2 = 0$

$\therefore$ $2R1 \Leftrightarrow 2 \cdot 2^2 - 3 \cdot 2 \cdot 1 + 1^2 = 3 \neq 0$.

So, $(2, 1) \notin R$ $\therefore$ R is not symmetric.

36. (b) : Since, side of square is of length 20 cm, therefore $x \in (0, 10)$.

37. (a) : Clearly, height of open box = x cm

Length of open box = $20 - 2x$

and width of open box = $20 - 2x$

$\therefore$ Volume (V) of the open box

$= x \times (20 - 2x) \times (20 - 2x)$

38. (d) : We have, $V = x(20 - 2x)^2$

$$\therefore \frac{dV}{dx} = x \cdot 2(20 - 2x)(-2) + (20 - 2x)^2$$

$= (20 - 2x)(-4x + 20 - 2x) = (20 - 2x)(20 - 6x)$

Now, $\frac{dV}{dx} = 0 \Rightarrow 20 - 2x = 0$ or $20 - 6x = 0$

$\Rightarrow x = 10$ or $\frac{10}{3}$

39. (c) : We have, $V = x(20 - 2x)^2$ and $\frac{dV}{dx}$

$= (20 - 2x)(20 - 6x)$

$$\Rightarrow \frac{d^2V}{dx^2} = (20 - 2x)(-6) + (20 - 6x)(-2)$$

$= (-2)[60 - 6x + 20 - 6x] = (-2)[80 - 12x] = 24x - 160$

For $x = \frac{10}{3}$, $\frac{d^2V}{dx^2} < 0$ and for $x = 10$, $\frac{d^2V}{dx^2} > 0$

So, volume will be maximum when $x = \frac{10}{3}$.

40. (d) : We have, $V = x(20 - 2x)^2$, which will be maximum when $x = \frac{10}{3}$.

$$\therefore \text{ Maximum volume } = \frac{10}{3}\left(20 - 2 \times \frac{10}{3}\right)^2$$

$$= \frac{10}{3} \times \frac{40}{3} \times \frac{40}{3} = \frac{16000}{27} \text{ cm}^3$$

41. (c) : Let B, R, Y and G denote the events that soft toy drawn is blue, red, yellow and green respectively.

$$\therefore P(B) = \frac{12}{35}, P(R) = \frac{8}{35}, P(Y) = \frac{10}{35} \text{ and } P(G) = \frac{5}{35}$$

$$P(B \cap G) = P(B) \cdot P(G \mid B) = \frac{12}{35} \cdot \frac{5}{34} = \frac{6}{119}$$

42. (b): $P(Y \cap R) = P(Y) \cdot P(R \mid Y) = \frac{10}{35} \cdot \frac{8}{34} = \frac{8}{119}$

43. (a) : Let E = event of drawing a first red soft toy and F = event of drawing a second red soft toy.

Here, $P(E) = \frac{8}{35}$ and $P(F \mid E) = \frac{7}{34}$

$$\therefore P(F \cap E) = P(E) \cdot P(F \mid E) = \frac{8}{35} \cdot \frac{7}{34} = \frac{4}{85}$$

44. (c) : $P(G \cap Y') = P(G) \cdot (Y' \mid G) = \frac{5}{35} \cdot \frac{24}{34} = \frac{12}{119}$

45. (d) : Let E = event of drawing a first non-blue soft toy and F = event of drawing a second non-blue soft toy

Here, $P(E) = \frac{23}{35}$ and $P(F|E) = \frac{22}{34}$

$\therefore \quad P(F \cap E) = P(E) \cdot P(F \mid E) = \frac{23}{35} \cdot \frac{22}{34} = \frac{253}{595}$

SECTION B2 (APPLIED MATHEMATICS)

16. (d) : Since A is a matrix of type 4×6, therefore, each row of A contains 6 elements. hence R is a row matrix of the type 1×6.

17. (b) : Given $x = 11$ km/h, $y = 6$ km/h

So, average speed $= \frac{(x+y)(x-y)}{x}$

$= \frac{(11+6)(11-6)}{11} = \frac{17 \times 5}{11} = 7.73$ km/h

18. (a) : To find the last digit of 12^{12}, we find 12^{12} (mod 10).

Since $12 \equiv 2 \pmod{10} \Rightarrow 12^4 \equiv 2^4 \pmod{10}$

$\Rightarrow \quad 12^4 \equiv 16 \pmod{10} \Rightarrow 12^4 \equiv 6 \pmod{10}$

$\Rightarrow \quad (12^4)^3 \equiv 6^3$ (mod 10)2

$\Rightarrow \quad 12^{12} \equiv 216 \pmod{10} \Rightarrow 12^{12} \equiv 6 \pmod{10}$.

Hence, the last digit of 12^{12} is 6.

19. (c) : Given, $x^4 + y^4 = 16$

Differentiate both sides w.r.t 'x', we get

$4x^3 + 4y^3 \frac{dy}{dx} = 0 \Rightarrow y^3 \frac{dy}{dx} = -x^3$ or $\frac{dy}{dx} = \frac{-x^3}{y^3}$

Again, differentiating w.r.t 'x' on both sides, we get

$y^3 \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 \times 3y^2 = -3x^2$

$\Rightarrow \quad y^3 \frac{d^2y}{dx^2} + \frac{x^6}{y^6} \cdot 3y^2 = -3x^2 \qquad \left[\because \frac{dy}{dx} = \frac{-x^3}{y^3}\right]$

$\Rightarrow \quad y^3 \frac{d^2y}{dx^2} + \frac{3x^6}{y^4} = -3x^2$

On multiplying both sides by y^4, we get

$y^7 \frac{d^2y}{dx^2} + 3x^6 = -3x^2 y^4$

$\Rightarrow \quad y^7 \frac{d^2y}{dx^2} = -3\left(16 - x^4\right)x^2 - 3x^6 \qquad \left[\because y^4 = 16 - x^4\right]$

$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{-48x^2}{y^7}$

20. (c) : We know that $\sum P_i = 1$

$\Rightarrow \quad 0.1 + 2k + 3k + 3k + 2k = 1$

$\Rightarrow \quad 10k = 1 - 0.1 \Rightarrow k = \frac{0.9}{10} = 0.09$

$\therefore$ Statement I is true.

Now, P(Ramesh study atleast two hours) $= P(X \geq 2)$

$= P(2) + P(3) + P(4) = 3k + 3k + 2k$

$= 8k = 8 \times 0.09 = 0.72$

P(Ramesh study atmost two hours) $= P(X \leq 2)$

$= P(X = 0) + P(X = 1) + P(X = 2) = 0.1 \times 2k + 3k$

$= 0.1 + 5k = 0.1 + 5 \times (0.09) = 0.1 + 0.45 = 0.55$

P(Ramesh studay exactly two hours) $= P(X = 2) = 3k$

$= 3 \times 0.09 = 0.27$

$\therefore$ Statement I and III are correct.

21. (d) : If price index is 118, then it represent that the prices have increased by 18%.

22. (a) : Original cost of machine = ₹ 45000

Scrap value of machine = ₹ 3000. Useful life = 15 years

$\therefore$ Annual depreciation $= \frac{45000 - 3000}{15} = \frac{42000}{15}$

= ₹ 2800

23. (c) : The values of $Z = px + qy$ at the points (3, 4) and (0, 5) are $3p + 4q$ and $5q$ respectively.

As Z has maximum value at both points (3, 4) and (0, 5), we get $3p + 4q = 5q \Rightarrow 3p = q$, which is the required relation between p and q.

24. (c) : Statement I is true but II is false as time reversal test is satisfied if $P_{01} \times P_{10} = 1$.

25. (b) : Given total profit = ₹ 74000

A's incentive = 5% of ₹ 74000 = ₹ $\frac{5}{100} \times 74000 = 3700$

Balance profit = ₹ 74000 – ₹ 3700 = ₹ 70300

Now, A's share of profit : B's share of profit : C's share of profit

$= 650000 \times 6 : 840000 \times 5 : 1000000 \times 3 = 13 : 14 : 10$

So, B's share in profit $= \frac{14}{37} \times$ ₹70300 = ₹26600 .

26. (b) : (P) Let $y = f(x)$, then $f(x) = x^3 - 2x^2 - 4x - 1$.

$f'(x) = 3x^2 - 4x - 4$ and

For extremum points, $f'(x) = 0 \Rightarrow 3x^2 - 4x - 4 = 0$

$\Rightarrow \quad (x - 2)(3x + 2) = 0 \Rightarrow x = 2, -\frac{2}{3}$ are critical points.

(Q) $f(x) = 5x^2 - 3x + 1\ f'(x) = 10x - 3$

For critical point, $f'(x) = 0$

$\Rightarrow \quad 10x - 3 = 0 \Rightarrow x = \frac{3}{10}$ is the critical point.

(R) $f(x) = x^2 - 2, f'(x) = 2x$

For critical point $f'(x) = 0$

$\Rightarrow \quad 2x = 0 \;\Rightarrow\; x = 0$ is the critical point.

(S) Let $f(x) = x + \frac{1}{x}, D_f = R - \{0\}$

It is differentiable for all $x \in D_f$; $f'(x) = 1 - \frac{1}{x^2}$.

Now, $f'(x) = 0 \Rightarrow 1 - \frac{1}{x^2} = 0 \Rightarrow x^2 = 1 \Rightarrow x = 1, -1$.

Therefore, the points where extremum may occur are $x = 1$ and $x = -1$.

27. (a) : $R = ₹\,7000,\; r = \frac{5.6}{4}\% = 1.4\%$. per quarter.

So, $i = \frac{1.4}{100} = 0.014$

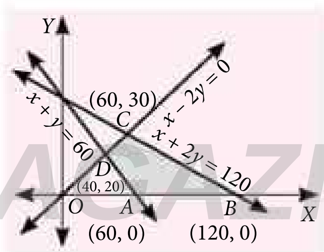
The given annuity is a perpetuity of second type.

So, $P = R + \frac{R}{i} = 7000 + \frac{7000}{0.014} = 7000 + 500000 = 507000$

Hence, ₹ 507000 is needed to invest.

28. (c) : Minimize $Z = 5x + 10y$ subject to the constraints $x + 2y \le 120$, $x + y \ge 60$, $x - 2y \ge 0$, $x \ge 0$, $y \ge 0$.

Draw the lines $x + 2y = 120$, $x + y = 60$, and $x - 2y = 0$; shade the region satisfies by the given inequalities. The feasible region is polygon $ABCD$, which is convex and bounded.



Corner points of feasible region are $A(60, 0)$, $B(120, 0)$, $C(60, 30)$ and $D(40, 20)$.

The values of $Z = 5x + 10y$ at the points A, B, C and D are 300, 600, 600 and 400 respectively.

Minimum value = 300 at A(60, 0).

29. (d) : Statement I is false, as the samples taken from the same population will differ, but will share the same characteristics of the population.

30. (a) : Let the length of the shortest piece of board be x cm, then the lengths of the second and third pieces are $(x + 3)$ cm and $2x$ cm respectively.

According to given,

$x + (x + 3) + 2x \le 87$ and $2x \ge (x + 3) + 5$

$\Rightarrow \quad 4x + 3 \le 87$ and $2x \ge x + 8 \Rightarrow 4x \le 84$ and $x \ge 8$

$\Rightarrow \quad x \le 21$ and $x \ge 8 \Rightarrow 8 \le x \le 21$.

31. (d) : We have, $C(x) = 48 + 15x + 3x^2$ and MC = AC

$\Rightarrow \quad \frac{d}{dx}\left(48 + 15x + 3x^2\right) = \frac{48 + 15x + 3x^2}{x}$

$\Rightarrow \quad 15 + 6x = \frac{48}{x} + 15 + 3x \Rightarrow \quad 3x = \frac{48}{x} \Rightarrow x^2 = \frac{48}{3}$

$\Rightarrow \quad x^2 = 16 \;\Rightarrow\; x = 4$ [taking only +ve value]

32. (c) : The number of ways of choosing two integers (without replacement) from the first five positive integers = ${}^5C_2 = 10$, so the sample space S has 10 equally likely outcomes. These outcomes are:

1, 2; 1, 3; 1, 4; 1, 5; 2, 3; 2, 4; 2, 5; 3, 4; 3, 5; 4, 5;

As the random variable X denotes the larger of the two numbers, X can take values 2, 3, 4, 5.

(Because 1 is not larger then any number from 1 to 5)

Note that in the sample space S, we have

Larger of two numbers	Number of outcomes
2	1
3	2
4	3
5	4

$P(X=2) = \frac{1}{10}, P(X=3) = \frac{2}{10}, P(X=4) = \frac{3}{10},$

$P(X=5) = \frac{4}{10}$

$\therefore$ The probability distribution of X is

X	2	3	4	5
$P(X)$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$

$\therefore \quad \text{Mean} = \mu = \sum p_i x_i = \frac{1}{10}(1\times 2 + 2\times 3 + 3\times 4 + 4\times 5)$

$= \frac{1}{10}(2 + 6 + 12 + 20) = \frac{40}{10} = 4.$

33. (b) : Given A runs 3 times as fast as B.

It means ratio of speeds of A and B = 3 : 1

$\Rightarrow$ When A covers 3 m, B covers 1 m

or we can say that A gains (3 – 1)m *i.e.* 2 m by B in a race of 3 m.

Given, A gives B a start of 50 m means A gains 50 m by B in the race.

Thus, when A gains 2 m by B, the race is of 3 m

when A gains 1 m by B, the race is of 3/2 m

when A gains 50 m by B, the race is of $\frac{3}{2} \times 50\,\text{m} = 75\,\text{m}$

Hence, length of race = 75 m.

34. (a) : Given, F = ₹2000, $r = \frac{8}{2} = 4\%$ per half year,

$N = 5 \times 2 = 10$ half years, $d = \frac{10}{2} = 5\% \;\Rightarrow\; i = 0.05$

So, $C = ₹2000 \times \frac{4}{100} = ₹80$

$$\text{P.V.} = \frac{80\left[1 - (1.05)^{-10}\right]}{0.05} + 2000 \times (1.05)^{-10}$$

$\therefore$ P.V. = $= \dfrac{80[1-0.6139]}{0.05} + 2000 \times 0.6139$

= 617.76 + 1227.8 = ₹ 1845.56.
Hence, the fair value of bond is ₹ 1845.56.

35. (d) : Singular matrix → Determinant is equal to zero
Identity matrix → Determinant is equal to one
Non square matrix → Determinant is not defined
Upper triangular → Determinant is equal to product of diagonal entries.

36. (b) : Let A alone fill the tank in t minutes, then B alone will fill the tank in $(t - 15)$ minutes and C alone will fill the tank in $(t - 15 - 5)$ *i.e* $(t - 20)$ minutes.

$\therefore$ According to given, $\dfrac{1}{t} + \dfrac{1}{t-15} = \dfrac{1}{t-20}$

$\Rightarrow \dfrac{t-15+t}{t(t-15)} = \dfrac{1}{t-20} \Rightarrow (2t-15)(t-20) = t(t-15)$

$\Rightarrow 2t^2 - 55t + 300 = t^2 - 15t$
$\Rightarrow t^2 - 40t + 300 = 0 \Rightarrow (t-10)(t-30) = 0$
$\Rightarrow t = 30, t = 10$ (neglecting) $(\because t > 20)$
Hence, the time required by pipe A to fill the tank = 30 minutes.

37. (c) : Let $u = 4x^3$ and $v = x^4$

By chain rule, we have $\dfrac{du}{dv} = \dfrac{du}{dx} \times \dfrac{dx}{dv}$

$\therefore \dfrac{d(4x^3)}{d(x^4)} = \dfrac{d(4x^3)}{dx} \times \dfrac{1}{\frac{d(x^4)}{dx}} = 12x^2 \times \dfrac{1}{4x^3} = \dfrac{3}{x}$

38. (a) : Cost of motorcycle = ₹ 150000
Down payment = ₹ 25000 $\therefore$ Balance = ₹ 125000

So, $P = 125000, \quad i = \dfrac{9}{12 \times 100} = 0.0075$
and $n = 4 \times 12 = 48$

$\therefore EMI = \dfrac{125000 \times 0.0075 \times (1.0075)^{48}}{(1.0075)^{48} - 1}$

$= \dfrac{125000 \times 0.0075 \times 1.4314}{0.4314}$ = ₹ 3110.66

39. (c) : Statement I and II are true because savings accounts doesn't necessarily have a specific purpose and sinking funds have smaller saving goals. Whereas saving account are meant for large unexpected expenses.

40. (b) : (29 + 42) mod 11
= 71 mod 11
So, 71 mod 11 = 5

$$\begin{array}{r|l} 11 & 71 \; (6 \\ & 66 \\ \hline & 5 \rightarrow \text{Remainder} \end{array}$$

41. (d) : First four years moving average

$= \dfrac{55+62+54+69}{4} = 60$

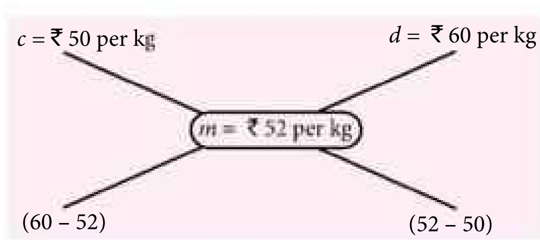
First 3 years moving average = $\dfrac{55+62+54}{3} = 57$

$\therefore$ Required difference = 60 – 57 = 3

42. (d) : Let the C.P. of the mixture be ₹ x per kg.
Then S.P. = C.P. + profit
$\Rightarrow 57.20 = x + 10\%$ of x

$\Rightarrow 57.20 = x + \dfrac{10x}{100} \Rightarrow 57.20 = \dfrac{11x}{10} \Rightarrow x = \dfrac{572}{11} \Rightarrow x = 52$

Now,



$\therefore \dfrac{\text{Quantity of cheaper}}{\text{Quantity of dearer}} = \dfrac{d-m}{m-c} = \dfrac{8}{2} = \dfrac{4}{1}$

Hence, the required ratio is 4 : 1.

43. (b) : $[x \quad 4 \quad -1] \begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 0 \\ 2 & 2 & 4 \end{bmatrix}$

$= [2x + 4 - 2 \quad x + 0 - 2 \quad -x + 0 - 4]$
$= [2x + 2 \quad x - 2 \quad -x - 4]$
Now, $[2x + 2 \quad x - 2 \quad -x - 4]\,[x \quad 4 \quad -1]^T$

$= [2x + 2 \quad x - 2 \quad -x - 4] \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix}$

$= [2x^2 + 2x + 4x - 8 + x + 4] = [2x^2 + 7x - 4]$
It is given that, $2x^2 + 7x - 4 = 0$
$\Rightarrow 2x^2 + 8x - x - 4 = 0$
$\Rightarrow 2x(x + 4) - 1(x + 4) = 0$

$\Rightarrow (2x - 1)(x + 4) = 0 \Rightarrow x = \dfrac{1}{2}$ or $x = -4$

Integral value of $x = -4$

44. (c) : $-8 \le 5x - 3 < 7$
$\Rightarrow -5 \le 5x < 10 \Rightarrow -1 \le x < 2$
$\therefore$ The solution set is $[-1, 2)$.

45. (c) : Seasonal variation

Quiz CLUB

Unlock Your Knowledge!

1. If $n(A) = 43$, $n(B) = 51$ and $n(A \cup B) = 75$, then how many elements are there in $(A - B) \cup (B - A)$?

2. Find the value of $\cos^2 75° + \cos^2 45° + \cos^2 15° - \cos^2 30° - \cos^2 60°$.

3. If $z = 1 + i$, then what will be the argument of $z^2 e^{z-i}$?

4. How many diagonals are there in a regular polygon of 100 sides?

5. If 'ω' is a complex cube root of unity, then find the value of $\omega^{\left(\frac{1}{3}+\frac{2}{9}+\frac{4}{27}+......\infty\right)} + \omega^{\left(\frac{1}{2}+\frac{3}{8}+\frac{9}{32}+......\infty\right)}$.

6. Find the eccentricity of the conic $x^2 - y^2 = (\sqrt{5})^{\frac{1}{10}}$.

7. Find the angle between the pair of lines $x^2 + 4xy + y^2 = 0$.

8. Evaluate : $\lim\limits_{x \to \frac{\pi}{4}} (5 - \tan x)^{\log \tan x}$

9. It mean of first 'n' odd natural numbers in $\frac{n^2}{36}$, then what will be the value of n?

10. A man, while dialling a telephone number, forgot the last two digits and remembering only that they are different, dialed them at random. What is the probability of the number being dialled correctly?

11. Find the range of the function $f(x) = 2|\sin x| - 3|\cos x|$.

12. Find principal value of $\cos^{-1}(\cos 5)$.

13. If the value of a third order determinant is 7, then what will be the value of square of determinant formed by its cofactors?

14. If A is a matrix of order $m \times n$ and B is a matrix such that AB' and $B'A$ and both defined, then what will be the order of matrix B?

15. If $x^2 + y^2 = 2$ and $y'' + Ay^{-3} = 0$, then find the value of A.

16. Find the value of 'a' for which the function $f(x) = x^2 + \frac{a}{x}$, has a minimum value at $x = 1$.

17. If $\hat{a}$ and $\hat{b}$ are two unit vectors and θ is the angle between them, then find the value of θ if $2\hat{b} + \hat{a}$ is a unit vector.

18. Find the value of 'a' if the plane $2x - 3y + 6z - 11 = 0$, makes an angle $\sin^{-1}(a)$ with x–axis.

19. Evaluate : $\int_{-\pi/4}^{\pi/4} \frac{(x^9 - 3x^5 + 7x^3 - x)}{\cos^2 x} dx$

20. Find the equation of curve whose slope at any point is thrice its abscissa and which passes through point $(-1, -3)$.

Readers can send their responses at editor@mtg.in or post us with complete address by 10th of every month. Winners' names and answers will be published in next issue.

beat the TIME TRAP

Duration : 60 minutes

SECTION-I

Single Option Correct Type

1. If $f(x)$ be an identity function, then

$$\lim_{x\to\infty}\left[\frac{f(x+1)f(x+2)+2}{f(x+3)f(x+4)}\right]^{4f(x)-2} \text{ equals}$$

(a) e^{-4} (b) e^{-16}
(c) e^{-12} (d) None of these

2. If $g(x) = x^2 + x - 1$ and

$(gof)(x) = 4x^2 - 10x + 5$, then $f\left(\frac{5}{4}\right)$ is equal to

(a) $\frac{-3}{2}$ (b) $\frac{-1}{2}$ (c) $\frac{1}{2}$ (d) $\frac{3}{2}$

3. The number of values of k, for which the system of equations $(k + 1)x + 8y = 4k$, $kx + (k + 3)y = 3k - 1$ has no solution, is

(a) 1 (b) 2
(c) 3 (d) Infinite

4. Let $f(x) = \begin{cases}\left[\tan\left(\frac{\pi}{4}+x\right)\right]^{1/x}, & x \neq 0\\ k, & x = 0\end{cases}$, then the value of k such that $f(x)$ hold continuity at $x = 0$ is

(a) e (b) e^3
(c) e^2 (d) none of these

5. If $f(x) = 3[x] + 5 = 5[x-2] + 7$, then $\int_1^2 x[x + f(x)]dx$ is equal to (where $[\cdot]$ represents G.I.F.)

(a) 63 (b) 63/2
(c) 126 (d) None of these

6. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$.
If vector $\vec{u}$ is perpendicular to vector $\vec{a}$ and $\vec{u}.\vec{b} = 24$, then $|\vec{u}|^2$ is equal to

(a) 336 (b) 315 (c) 256 (d) 84

7. Consider 5 independent Bernoulli's trials, each with probability of success p. If the probability of at least one failure is greater than or equal to 31/32, then p lies in the interval

(a) $\left[0, \frac{1}{2}\right]$ (b) $\left(\frac{11}{12}, 1\right]$
(c) $\left(\frac{1}{2}, \frac{3}{4}\right]$ (d) $\left(\frac{3}{4}, \frac{11}{12}\right]$

8. The equation of the plane through the line of intersection of planes $ax + by + cz + d = 0$, $a'x + b'y + c'z + d' = 0$ and parallel to the lines $y = 0 = z$ is

(a) $(ab' - a'b)x + (bc' - b'c)y + (ad' - a'd) = 0$
(b) $(ab' - a'b)y + (ac' - a'c)z + (ad' - a'd) = 0$
(c) $(ab' - a'b)x + (bc' - b'c)z + (ad' - a'd) = 0$
(d) none of these

9. If $x = 111....1$ (20 digits), $y = 333...3$ (10 digits) and $z = 222....2$ (10 digits), then $\frac{x - y^2}{z} =$

(a) 1 (b) 2 (c) 1/2 (d) 3

10. The solution of the equation $(3|x| - 3)^2 = |x| + 7$ which belongs to the domain of definition of the function $y = \sqrt{x(x-3)}$ are given by

(a) $\pm\frac{1}{9}, \pm 2$ (b) $-\frac{1}{9}, 2$
(c) $\frac{1}{9}, -2$ (d) $-\frac{1}{9}, -2$

SECTION-II

Numerical Answer Type

11. If the real part of the complex number $(1 - \cos\theta + 2i\sin\theta)^{-1}$ is 1/5, for $\theta \in (0, \pi)$, then the value of the integral $\int_0^\theta \sin x\, dx$ is equal to __________.

12. In a plane, there are two families of lines $y = x + r$ and $y = -x + r$, where $i \in [0, 4]$. The number of squares of diagonals of the length 2 unit formed by the lines are/is_______.

13. For $y > 0$ and $x \in R$, $e^x y dx - y^2 dy = e^x dy$, where $y = f(x)$. If $f(\log 2) = 1$, then $f(\log 6)$ equals______.

14. Let $f(x) = ax^2 - 8x + b$ and $3^{\sqrt{\log_3 7}}$ and $7^{\sqrt{\log_7 3}}$ are roots of the equation $f(x) = 0$, then maximum value of $f(1)$ is, (If a, b are positive integers)______.

15. Let A be a 5×5 matrix such that $\det(A) = -3$, then $\det(-3A^{-1}) + 3\det(A)$ equals_______.

SOLUTIONS

1. **(b) :** As $f(x)$ is an identity function

$f(x) = x \quad \therefore f(x + \alpha) = x + \alpha,\ \alpha \in R$

$$\therefore \lim_{x\to\infty}\left(\frac{f(x+1)f(x+2)+2}{f(x+3)f(x+4)}\right)^{4f(x)-2}$$

$$= \lim_{x\to\infty}\left(\frac{(x+1)(x+2)+2}{(x+3)(x+4)}\right)^{(4x-2)}$$

$$= \lim_{x\to\infty}\left(\frac{x^2+3x+4}{x^2+7x+12}\right)^{4x-2} \quad \text{(which is } 1^\infty \text{ form)}$$

$$= e^{\lim_{x\to\infty}\frac{-(4x+8)(4x-2)}{x^2+7x+12}} = e^{-16}$$

2. **(b) :** Since, $g(x) = x^2 + x - 1$

$\therefore \quad (gof)(x) = g(f(x)) = f^2(x) + f(x) - 1$

$$\Rightarrow g\left(f\left(\frac{5}{4}\right)\right) = f^2\left(\frac{5}{4}\right) + f\left(\frac{5}{4}\right) - 1 \quad \text{...(i)}$$

$$\text{Again, } g\left(f\left(\frac{5}{4}\right)\right) = 4\left(\frac{5}{4}\right)^2 - 10\left(\frac{5}{4}\right) + 5 = -\frac{5}{4} \quad \text{...(ii)}$$

From (i) and (ii), we get $-\frac{5}{4} = f^2\left(\frac{5}{4}\right) + f\left(\frac{5}{4}\right) - 1$

$$\Rightarrow f^2\left(\frac{5}{4}\right) + f\left(\frac{5}{4}\right) + \frac{1}{4} = 0$$

$$\Rightarrow \left(f\left(\frac{5}{4}\right) + \frac{1}{2}\right)^2 = 0 \Rightarrow f\left(\frac{5}{4}\right) = -\frac{1}{2}$$

3. **(c) :** The equation is $\begin{bmatrix} k+1 & 8 \\ k & k+3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4k \\ 3k-1 \end{bmatrix}$

For no solution of $AX = B$, a necessary condition is $\det A = 0$.

$$\Rightarrow \begin{vmatrix} k+1 & 8 \\ k & k+3 \end{vmatrix} = 0 \Rightarrow (k+1)(k+3) - 8k = 0$$

$\Rightarrow k^2 + 4k + 3 - 8k = 0 \Rightarrow k^2 - 4k + 3 = 0$

$\Rightarrow (k-1)(k-3) = 0 \quad \therefore \quad k = 1, 3$

For $k = 1$, the equation becomes

$2x + 8y = 4,\ x + 4y = 2$

which is just a single equation in two variables.

$\therefore \quad x + 4y = 2$ has infinite solutions.

For $k = 3$, the equation becomes

$4x + 8y = 12,\ 3x + 6y = 8$

which are parallel lines. So no solution in this case.

4. **(c) :** $\underset{x\to 0}{\text{Lt}}\ f(x) = \underset{x\to 0}{\text{Lt}}\left[\tan\left(\frac{\pi}{4} + x\right)\right]^{1/x}$

$$= \underset{x\to 0}{\text{Lt}}\left(\frac{1+\tan x}{1-\tan x}\right)^{1/x}$$

$$= \underset{x\to 0}{\text{Lt}}\left\{(1+\tan x)^{\frac{1}{\tan x}}\right\}^{\frac{\tan x}{x}} \times \underset{x\to 0}{\text{Lt}}\left\{(1-\tan x)^{-\frac{1}{\tan x}}\right\}^{\frac{\tan x}{x}}$$

$= e \times e \quad \therefore k = e^2$

5. **(b) :** $f(x) = 3[x] + 5 = 5[x-2] + 7$

$\Rightarrow \quad 3[x] + 5 = 5[x] - 10 + 7 \Rightarrow 2[x] = 8 \quad \therefore \quad [x] = 4$

$\Rightarrow \quad x \in [4, 5)$ *i.e.* $4 \le x < 5 \qquad \therefore x = 4 +$ fractional part

$\therefore \quad f(x) = 3[x] + 5 = 5[x-2] + 7$

$\therefore \quad f(4) = 12 + 5 = 17$

$\therefore \quad [x + f(x)] = [4 + \text{fractional part} + 17] = 21$

$$\text{Now, } \int_1^2 x[x + f(x)]dx = \int_1^2 21x\,dx = \frac{63}{2}$$

6. **(a) :** Let $\vec{u} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{u}\cdot\vec{a} = 0$ gives $2x + 3y - z = 0$ ...(i)

$\vec{u}\cdot\vec{b} = 24$ gives $y + z = 24$ ...(ii)

Also $\vec{u}$ is coplanar with $\vec{a}$ and $\vec{b}$, so $\left[\vec{u}\ \vec{a}\ \vec{b}\right] = 0$, which

$$\text{yields } \begin{vmatrix} x & y & z \\ 2 & 3 & -1 \\ 0 & 1 & 1 \end{vmatrix} = 0 \quad i.e.\ \ 4x - 2y + 2z = 0 \quad \text{...(iii)}$$

(ii) and (iii) gives $2x + 2z = 24$ *i.e.* $x + z = 12$ ...(iv)

From (i), and (ii) gives $x - 2z = -36$ ...(v)

On solving (iv) and (v) we get

$z = 16$ and thus $x = -4$ and $y = 8$.

Hence, $|\vec{u}| = \sqrt{(-4)^2 + 8^2 + 16^2} = \sqrt{336}$

$\therefore \ |\vec{u}|^2 = 336$

7. **(a) :** Probability of at least one failure

$= 1 - P(\text{no failure}) = 1 - p^5$

$$\text{Now, } 1 - p^5 \ge \frac{31}{32}$$

$$\Rightarrow \quad p^5 \le \frac{1}{32} \text{ thus } p \le \frac{1}{2} \quad \therefore \ p \in [0, 1/2]$$

8. **(b) :** Equation of planes passes through the intersection of the planes $ax + by + cz + d = 0$

and $a'x + b'y + c'z + d' = 0$ is

$(a'x + b'y + c'z + d') + \lambda(ax + by + cz + d) = 0$ ...(i)

which is parallel to $y = 0 = z$

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means parallel to x-axis

$\therefore \quad (a' + a\lambda)\, 1 + 0(b' + b\lambda) + 0(c' + c\lambda) = 0$

$\Rightarrow \quad a\lambda = -a' \quad \therefore \quad \lambda = -\dfrac{a'}{a}$

Putting $\lambda = -\dfrac{a'}{a}$ in (i), we have

$a(a'x + b'y + c'z + d') - a'(ax + by + cz + d) = 0$

$\Rightarrow a'x + ab'y + ac'z + ad' - aa'x - a'by - a'cz - a'd = 0$

$\Rightarrow (ab' - a'b)y + (ac' - a'c)z + ad' - a'd = 0$

9. (a) : $x = \dfrac{1}{9}(999\ldots9) = \dfrac{1}{9}(10^{20} - 1)$

$y = \dfrac{1}{3}(999\ldots9) = \dfrac{1}{3}(10^{10} - 1)$

$z = \dfrac{2}{9}(999\ldots9) = \dfrac{2}{9}(10^{10} - 1)$

$\therefore \dfrac{x - y^2}{z} = \dfrac{10^{20} - 1 - (10^{10} - 1)^2}{2(10^{10} - 1)} = \dfrac{10^{10} + 1 - (10^{10} - 1)}{2} = 1$

10. (d) : Domain of definition of the function

$y = \sqrt{x(x-3)}$ is given by $x(x - 3) \geq 0$

$\Rightarrow \quad x \geq 0,\ x - 3 \geq 0$ or $x \leq 0,\ x \leq 3 \ \Rightarrow\ x \leq 0$ or $x \geq 3$

Again given equation can be written as

$9|x|^2 - 18|x| + 9 = |x| + 7$

$\Rightarrow \quad 9|x|^2 - 19|x| + 2 = 0$

$\Rightarrow \quad 9|x|^2 - 18|x| - |x| + 2 = 0$

$\Rightarrow \quad 9|x|(|x| - 2) - 1(|x| - 2) = 0$

$\Rightarrow \quad (9|x| - 1)(|x| - 2) = 0 \ \Rightarrow |x| = 1/9$ or $|x| = 2$

$\Rightarrow \quad x = \pm\dfrac{1}{9}$ or $x = \pm 2$. But $x \leq 0$ or $x \geq 3$

Hence the required solutions are $x = -2, -1/9$

11. (1) : Let $z = (1 - \cos\theta + 2i\sin\theta)^{-1}$, then

$$z = \frac{1}{(1 - \cos\theta + 2i\sin\theta)} \times \frac{1 - \cos\theta - 2i\sin\theta}{1 - \cos\theta - 2i\sin\theta}$$

$$= \frac{1 - \cos\theta - 2i\sin\theta}{(1 - \cos\theta)^2 + 4\sin^2\theta} = \frac{2\sin^2\frac{\theta}{2} - 4i\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}{(1 - \cos\theta)^2 + 4\sin^2\theta}$$

$$= \frac{\sin\frac{\theta}{2} - 2i\cos\frac{\theta}{2}}{2\sin\frac{\theta}{2}\left(\sin^2\frac{\theta}{2} + 4\cos^2\frac{\theta}{2}\right)},$$

$$\Rightarrow \quad \text{Re}(z) = \frac{1}{2\left(\sin^2\frac{\theta}{2} + 4\cos^2\frac{\theta}{2}\right)} = \frac{1}{5}$$

$\Rightarrow \quad \sin^2\dfrac{\theta}{2} + 4\cos^2\dfrac{\theta}{2} = \dfrac{5}{2}$ (Given)

$\Rightarrow \quad 1 - \cos^2\dfrac{\theta}{2} + 4\cos^2\dfrac{\theta}{2} = \dfrac{5}{2}$

$\Rightarrow \quad 3\cos^2\dfrac{\theta}{2} = \dfrac{3}{2} \ \Rightarrow\ \cos^2\dfrac{\theta}{2} = \dfrac{1}{2}$

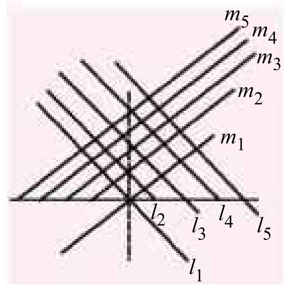
$\Rightarrow \quad \cos\dfrac{\theta}{2} = \dfrac{1}{\sqrt{2}}$ $\quad [\because\ \theta \in (0, \pi) \Rightarrow 0 < \theta/2 < \pi/2]$

$\Rightarrow \quad \dfrac{\theta}{2} = \dfrac{\pi}{4} \Rightarrow \theta = \pi/2 \ \therefore\ \int_0^{\pi/2} \sin x\, dx = [-\cos x]_0^{\pi/2} = 1$

12. (9) : As $i = 0, 1, 2, 3, 4$

$\therefore \quad n = 5$

which means there are two sets of 5 parallel lines. Now the set of lines (l_1, l_3) and (m_1, m_3), (l_2, l_4) and (m_2, m_4) and (l_3, l_5) and (m_3, m_5) form the required squares.



$\therefore \quad$ Required number of squares are $3 \times 3 = 9$.

13. (2) : Given, $e^x y\,dx - y^2 dy = e^x dy$

$\Rightarrow \quad e^x(y\,dx - dy) = y^2 dy$

$\Rightarrow \quad e^x\left(\dfrac{y\,dx - dy}{y^2}\right) = dy \Rightarrow d\left(\dfrac{e^x}{y}\right) = dy$

$\Rightarrow \quad \dfrac{e^x}{y} = y + c$ ...(i)

At $x = \log 2$, $y = 1$ we get $c = 1$

$\therefore \quad$ (i) becomes, $e^x = y(y + 1)$

For $f(\log 6)$, we have

$y(y + 1) - 6 = 0 \Rightarrow y^2 + y - 6 = 0 \Rightarrow (y + 3)(y - 2) = 0$

$\therefore \quad y = 2$ as $y > 0$

14. (9) : Given, roots of $f(x) = 0$ are $3^{\sqrt{\log_3 7}}$ and $7^{\sqrt{\log_7 3}}$, which are equal (same valued number)

$\therefore \quad D = 0$

$\Rightarrow \quad 4ab = 64$

$\therefore \quad ab = 16$ ...(i)

Again, $f(1) = a + b - 8$ ...(ii)

Now, set of numbers whose product is 16 are (4, 4), (16, 1), (1, 16), (2, 8), (8, 2) but by inspection, we note that (1, 16) & (16, 1) are suitable sets for $f(1)$ to be maximum, then these sets are in our consideration.

$\therefore \quad f(1) = 1 + 16 - 8 = 9$, which is maximum value of $f(1)$

15. (72) : As $\det(A) = -3 \neq 0$

$\therefore \quad A^{-1}$ exist and the order of A^{-1} be also 5×5

Now, $\det(-3A^{-1}) + 3\det(A)$

$= (-3)^5 \det(A^{-1}) + 3 \times (-3) = (-3)^5 (\det(A))^{-1} - 9$

$= (-3)^5 \cdot \dfrac{1}{\det(A)} - 3^2 = (-3)^5\left(-\dfrac{1}{3}\right) - 3^2 = 3^2(3^2 - 1) = 72$

UCD
Unique Career in Demand

Explore the available Unique Career Options!

ROBOTICS ENGINEERING

Robotics Engineering is the field of engineering that deals with building machines that replicate human actions. These applications or autonomous machines (robots) are created by a robotics engineer for industries including mining, manufacturing, automotive, services and more. The purpose is to programme the machines to do repetitive, hazardous or unhealthy jobs.

Work of a Robotics Engineer

The work of a robotics engineer is to design prototypes, build and test machines and maintain the software that controls them. On the other hand, they also do research to find out the most cost-efficient and safest process to manufacture their robotics systems .

Courses Offered

1. Diploma Course

An interested aspirants who are looking for short-term courses in Robotics Engineering can join this diploma course. Its duration is upto 3 years.

- To pursue this course, an aspirant needs to complete their 10th class from a recognised board.
- There are a few colleges in India that provide diplomas in Robotics Engineering. Aspirants who want to pursue this course can choose between Electrical and Electronics Engineering or a diploma in Computer Science Engineering course.

2. Undergraduate Degree

- The undergraduate degree programme in Robotics Engineering is B.Tech. To pursue this course, an aspirant needs to complete their class 12th with Physics, Chemistry and Mathematics subjects.
- The duration of this course is 4 years.
- To pursue this course, an aspirant needs to crack entrance exams that are conducted by colleges or universities.
- In various colleges, this specialisation is provided with the combination of Automation or Electronics, like B.Tech in Automation and Robotics Engineering and B.Tech in Electronics and Robotics Engineering.

3. Postgraduate Course

- It is a master's degree course and the degree provided in this course is M.Tech.
- Its duration is two years.
- To pursue this course, aspirants need to crack the GATE exam.
- This degree course provides in-depth knowledge in the areas of Robotics Engineering by covering Electro-Mechanics, Robotics Sensors and Instrumentation, Robotics Fabrication, Artificial Intelligence and Robotics Vision.

B.Tech in Robotics Engineering : Top Exams

JEE Main: It is conducted by NTA twice a year. JEE is one of the major engineering entrance exams in the country for seeking admission into the most reputed institutes for undergraduate engineering and architecture programmes in IITs, NITs and other CFTIs funded by participating state governments, etc.

JEE Advanced: It is conducted by the Indian Institute of Technology once a year. Aspirants who clear the JEE Main exam are only eligible to appear for the JEE Advanced examination. Aspirants who want to study and take admission to IITs need to clear this exam.

SRMJEEE: This is a university level exam which is conducted by the SRM Institute of Science and Technology for taking admission in B.Tech from across its campuses.

BITSAT: This exam is conducted by the Birla Institute of Technology and Science.

Jobs and Career Prospects

Those aspirants who are pursuing a Robotics Engineering degree can grab the job opportunities available in Humanoid Robot Programming, Construction, Healthcare, Automated Management, Maintenance, etc. There are several positions available in India as well as outside India. There is a huge demand for Robotics Engineering in the gaming industry. The robotic revolution is about to take root in the era of the industrial revolution. That is why a tremendous increase has been seen in the sector of Robotic Engineering in the past few years. The robotic sector is growing rapidly with each passing advancement. A few full-time job titles in Robotics Engineering are :

- Robotics Specialist
- Robotics Technician
- Mobile Robotics Application Engineer
- Robotics Application Engineer
- Lead Robotics Software Engineer
- Senior Robotics Specialist
- Aerospace Robotics Engineer

List of Robotics Engineering Colleges in India

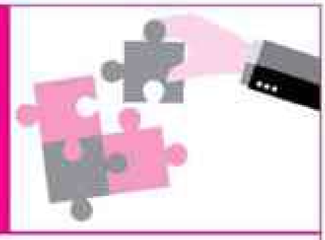
Some of the Robotics Engineering colleges in India are :

- IIIT Allahabad - Indian Institute of Information Technology
- ITM Vocational University
- Rajalakshmi Engineering College
- IIT Roorkee - Indian Institute of Technology
- Jadavpur University
- Manipal Institute of Technology
- Vellore Institute of Technology, Chennai, VIT University
- KL College of Engineering, KL University
- IGDTUW - Indira Gandhi Delhi Technical University For Women
- MSU - The Maharaja Sayajirao University of Baroda
- GGSIPU - Guru Gobind Singh Indraprastha University

Robotics Engineering Top Recruiters

- TATA
- DRDO
- ISRO
- BHEL
- NASA
- Precision Automation and Robotics India ltd.
- Difacto Robotics and Automation
- Kuka Robotics
- Tech Mahindra Ltd.

PUZZLE CORNER



ANSWER - APRIL 2023

12+ 4	11+ 5	9× 1	3	3÷ 2	6
5	6	3	2÷ 4	2÷ 1	2
3	3− 1	4 4	2	12+ 6	5
11+ 2	4	90× 5	6	3	1
6	3	3÷ 2	25× 1	5	48× 4
2÷ 1	2	6	5	4	3

GK CORNER

Enhance Your General Knowledge with Current Updates!

AWARDS AND RECOGNITIONS

- The Academy Awards or Oscars Awards 2023, which were originally held in 1929, at the Hollywood Roosevelt Hotel, recently celebrated their 95th anniversary. The Oscars Awards 2023 held on March 12 at the Dolby Theatre in Los Angeles.
  - The song 'Naatu Naatu' from the hit Telugu-language film RRR composed by M.M. Keeravani, with lyrics by chandrabose and recorded by Rahul Sipligunj and Kaala Bhairava, has made history by becoming the first Indian film song to win an Oscar for "Best Original Song".
- Aaliya Mir, Education Officer and Programme Head, was felicitated with the Wildlife Conservation Award by Jammu and Kashmir, becoming the first woman in the Union Territory for her conservation efforts in the region. Aaliya Mir is also Kashmir's first woman who works for the Charity Wildlife SOS Organisation, which is part of the Wildlife Rescue Team.
- SAARC Literary Award is an annual award conferred by the Foundation of SAARC Writers and Literature (FOSWAL) since 2001.Shamshur Rahman, Mahasweta Devi, Jayanta Mahapatra, Abhi Subedi, Mark Tully, Sitakant Mahapatra, Uday Prakash, Suman Pokhrel and Abhay K have been some of the recipients of this award. Nepali poet, lyricist and translator Suman Pokhrel is only writer to have been given this award twice.
- The Abel Prize is awarded annually to outstanding International Mathematicians. The prize was established by the Norwegian Government in 2002 and is managed by the Norwegian Academy of Science and Letters. Luis A. Caffarelli of the University of Texas at Austin, USA awarded with 2023, Abel Prize.
- Tamil writer Sivasankari has been selected for the Saraswati Samman for the year 2022 for her Memoir (Book) Survya Vamsam. The Saraswati Samman is one of the three literary awards instituted by the KK Birla Foundation in 1991. It is the highest recognition in the field of Indian literature in the country and carries a cash prize of ₹15 lakh, a Plaque and a Citation.
- The Vyas Samman is the second Highest Literacy Award, after the Jnanpith Award. The annual Vyas Samman is given to a superb piece of Indian Hindi literature, published within the previous ten years by the K.K. Birla Foundation. Hindi writer Dr. Gyan Chaturvedi's 2018 satirical novel, Pagalkhana, has been selected for the 32nd Vyas samman, 2022.
- British architect Sir David Alan Chipperfield, has been awarded 52nd architecture's top award, the Pritzker Architecture Prize for 2023. The award is regarded internationally as architecture's highest honor.
- Weightlifter Saikhom Mirabai Chanu has won the 2022 'BBC Indian Sportswoman of the Year' award after a public vote. Chanu becomes the first athlete to win the award twice in a row after winning for the year 2021.
- Delhi's Indira Gandhi International Airport has been adjudged as one of the best and cleanest airport in the Asia Pacific region by International Grouping (ACI), a non-profit organisation of airport operators.
- Managing Director (MD) and Chief Executive Officer of HDFC Bank, Sashidhar Jagdishan has been chosen as the "Business Standard Banker of the Year 2022". HDFC has continued its strong performance under Sashidhar's leadership while successfully navigating through technology-related challenges.
- Groupe Speciale Mobile Association (GSMA) has conferred Government Leadership Award, 2023 to India for implementing best practices in telecom

policy and regulation. GSMA, which represents more than 750 mobile operators and 400 companies in the telecom ecosystem, recognizes one country every year. India was declared winner in the ceremony held at Mobile World Congress Barcelona.

- Bharat Heavy Electricals Limited (BHEL) has been awarded the CBIP Award 2022 for 'Best Contribution in Solar Energy.' CBIP awards are conferred for outstanding contribution to the development of water, power and renewable energy sectors.

Test Yourself!

1. Which of the following Maharatna company awarded the "CBIP Award, 2022" for 'Best Contribution in Solar Energy'?
   (a) Gas Authority of India Ltd. (GAIL)
   (b) Bharat Heavy Electricals Limited (BHEL)
   (c) Engineers India Limited (EIL)
   (d) National Mineral Development Corporation (NMDC)

2. Which female sportsperson becomes the first athlete to win the award twice in a row?
   (a) Mithali Raj (b) PV Sindhu
   (c) Mirabai Chanu (d) Jhulan Goswami

3. As per the Airports Council International (ACI) ranking, which airport is one of the best and cleanest airports in the Asia Pacific region?
   (a) Chattrapati Shivaji International Airport
   (b) Rajiv Gandhi International Airport
   (c) Chennai International Airport
   (d) Indira Gandhi International Airport

4. Which among the following has been chosen as the 'BS Banker of the Year 2022?
   (a) Sashidhar Jagdishan
   (b) Kishore Kumar Poludasu
   (c) Shri Dinesh Kumar Khara
   (d) Sandeep Bakhshi

5. Which of the following was awarded with Architecture's top award, the Pritzker Prize, 2023?
   (a) Naveen Jindal
   (b) Tan Twan Eng
   (c) Janice Pariat
   (d) Sir David Alan Chipperfield

6. Select the correct statement regarding "Vyas Samman, 2023".
   (a) The KK Birla Foundation founded the annual Vyas Samman in 2012.
   (b) Arundhati Roy's *"pagalkhana"* was chosen for the prestigious Vyas Samman.
   (c) Hindi author Dr. Gyan Chaturvedi, has been chosen for the 32nd Vyas Samman.
   (d) All of the above.

7. Luis A. Caffarelli, has won the 2023 'Abel Prize'. This award is related to which category?
   (a) Mathematics (b) Chemistry
   (c) Biology (d) Physics

8. Who is the director of the song which has been shortlisted for the 2023 Oscar Awards in the 'Best Original Song' category?
   (a) Yuvan Shankar Raja
   (b) M.M. Keeravani
   (c) Anirudh Ravichander
   (d) Gangai Amaren

9. Which among the following is the only writer to have been given the SAARC Literary Award twice?
   (a) G. Sankara Kurup (b) Abhi Subedi
   (c) Suman Pokhrel (d) Sitakant Mahapatra

10. Which movie recently won four Hollywood Critics Association Film Awards 2023?
   (a) Ram Setu
   (b) A Knives Out Mystery
   (c) Viking Wolf
   (d) RRR

Answer Key

1.	(b)	2.	(c)	3.	(d)	4.	(a)	5.	(d)
6.	(c)	7.	(a)	8.	(b)	9.	(c)	10.	(d)

EXAM ALERT 2023

Exam	Date
MHT CET PCM	9th to 13th May
KCET BM	20th May
KCET PC	21st May
CUET	21st to 31st May
KCET Language Test	22nd May
BITSAT Session I	22nd to 26th May
JEE Advanced	4th June
BITSAT Session II	18th to 22nd June

Are You Ready for JEE?

Top Weightage Topics for JEE

In this article we will discuss about some of the most important topics of mathematics in respect to JEE Main and Advanced. It is based on the analysis of previous years' questions of JEE Main and Advanced.

SERIES 3

Limits

The value that a function approaches as that functions input get closer and closer to some number.

Definition

Limit of a function $y = f(x)$ exists at $x = a$, iff L.H.L. = R.H.L. where L.H.L = $\lim_{x \to a^-} f(x)$ means limit approaching curve at $x = a$ to the left of a and R.H.L = $\lim_{x \to a^+} f(x)$ means limit approaching curve at $x = a$ to the right of a.

Bounded Function

A function f is said to be bounded in $]a, b[$ if there exist some real numbers K_1 and K_2 such that

$K_1 \le f(x) \le K_2 \ \forall \ x \in \ ]a, b[$.

L' Hospital's Rule

Let $f(a) = 0$, $g(a) = 0$ and $f(x)$, $g(x)$ are differentiable functions with derivatives $f'(x)$, $g'(x)$, then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f(x) - f(a)}{g(x) - g(a)} = \lim_{x \to a} \frac{\frac{f(x) - f(a)}{x - a}}{\frac{g(x) - g(a)}{x - a}} = \frac{\lim_{x \to a} \frac{f(x) - f(a)}{x - a}}{\lim_{x \to a} \frac{g(x) - g(a)}{x - a}} = \frac{f'(a)}{g'(a)}, \ g'(a) \ne 0$$

or $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)} = \frac{f''(a)}{g''(a)}$ if $f'(a) = g'(a) = 0$ and so on.

Theorem for Evaluation of Limits

If $\underset{x \to a}{\text{Lt}} f(x)$ and $\underset{x \to a}{\text{Lt}} g(x)$ exist, then

- $\underset{x \to a}{\text{Lt}} [f(x) \pm g(x)] = \underset{x \to a}{\text{Lt}} f(x) \pm \underset{x \to a}{\text{Lt}} g(x)$
- $\underset{x \to a}{\text{Lt}} [f(x) \cdot g(x)] = \underset{x \to a}{\text{Lt}} f(x) \cdot \underset{x \to a}{\text{Lt}} g(x)$
- $\underset{x \to a}{\text{Lt}} f(x)^n = \left(\underset{x \to a}{\text{Lt}} f(x)\right)^n = l^n$,

  provided $\underset{x \to a}{\text{Lt}} f(x) = l$ and n is a natural number.
- $\underset{x \to a}{\text{Lt}} \frac{f(x)}{g(x)} = \frac{\underset{x \to a}{\text{Lt}} f(x)}{\underset{x \to a}{\text{Lt}} g(x)}$ $\left(\text{provided } \underset{x \to a}{\text{Lt}} g(x) \ne 0\right)$
- $\underset{x \to a}{\text{Lt}} \left(\frac{1}{f(x)}\right) = \frac{1}{\underset{x \to a}{\text{Lt}} f(x)} = \frac{1}{l}$ (provided $l \ne 0$ and $\underset{x \to a}{\text{Lt}} f(x) = l$)
- If $\underset{x \to a}{\text{Lt}} f(x) = c$ and $\underset{x \to c}{\text{Lt}} g(x) = l$, then $\underset{x \to a}{\text{Lt}} g(f(x)) = l$
- $\underset{x \to a}{\text{Lt}} |f(x)| = \left|\underset{x \to a}{\text{Lt}} f(x)\right| = |l|$, provided $\underset{x \to a}{\text{Lt}} f(x) = l$
- $\underset{x \to a}{\text{Lt}} e^{f(x)} = e^{\underset{x \to a}{\text{Lt}} f(x)}$
- If $\underset{x \to a}{\text{Lt}} f(x) = A > 0$, $\underset{x \to a}{\text{Lt}} g(x) = B$, a finite number, then

  $\underset{x \to a}{\text{Lt}} f(x)^{g(x)} = e^{\underset{x \to a}{\text{Lt}} g(x) \cdot \log_e f(x)}$

  $= e^{\underset{x \to a}{\text{Lt}} g(x) \ \underset{x \to a}{\text{Lt}} \log_e (f(x))} = e^{B \log_e A} = A^B$

Evaluation of Limits when $X \to \infty$

Let the given expression is $\frac{f(x)}{g(x)}$.

In such cases, we note the degree of $f(x)$ and $g(x)$ and then divide the numerator and denominator by the highest power of the variable term. Further we use $\underset{x \to \infty}{\text{Lt}} \frac{1}{x^n} = 0 \ (x > 0, \neq 1)$

Evaluation of Limits of the form 1^{∞}

- If $\underset{x \to a}{\text{Lt}} f(x) = 0 = \underset{x \to a}{\text{Lt}} g(x)$, then

  $$\underset{x \to a}{\text{Lt}} [1 + f(x)]^{\frac{1}{g(x)}} = e^{\underset{x \to a}{\text{Lt}} [1 + f(x) - 1] \times \frac{1}{g(x)}} = e^{\underset{x \to a}{\text{Lt}} \frac{f(x)}{g(x)}}$$

- If $\underset{x \to a}{\text{Lt}} f(x) = 0$ and $\underset{x \to a}{\text{Lt}} g(x) = \infty$, then

  $\underset{x \to a}{\text{Lt}} [1 + f(x)]^{g(x)}$ (yield 1^{∞} form) $= e^{\underset{x \to a}{\text{Lt}} f(x) \times g(x)}$

Some Standard Results on Limits

- $\underset{x \to 0}{\text{Lt}} \frac{\sin x}{x} = 1 = \underset{x \to \infty}{\text{Lt}} x \sin(1/x)$
- $\underset{x \to 0}{\text{Lt}} \frac{\tan x}{x} = 1 = \underset{x \to \infty}{\text{Lt}} x \tan(1/x)$
- $\underset{x \to 0}{\text{Lt}} \frac{\sin ax}{x} = \underset{x \to 0}{\text{Lt}} \frac{\tan ax}{x} = a$
- $\underset{x \to 0}{\text{Lt}} \frac{a^x - 1}{x} = \ln a$
- $\underset{x \to 0}{\text{Lt}} \frac{e^x - 1}{x} = 1$
- $\underset{x \to a}{\text{Lt}} \log x = \log a$, provided $a > 0$
- $\underset{x \to 0}{\text{Lt}} \frac{\log_e(1 + x)}{x} = \underset{x \to 0}{\text{Lt}} = 1$
- $\underset{x \to 0}{\text{Lt}} \frac{\ln(1 + ax)}{x} = a$
- $\lim_{x \to a} \frac{x^m - a^m}{x^n - a^n} = \frac{m}{n} a^{m-n}$
- $\lim_{x \to a} \frac{\sin^{-1} x}{x} = 1 = \lim_{x \to 0} \frac{\tan^{-1} x}{x}$
- If $g(x) \leq f(x) \leq h(x) \ \forall \ x \in]b - k, b + k[$ and $\underset{x \to b}{\text{Lt}} g(x) = \underset{x \to b}{\text{Lt}} h(x) = l$, then $\underset{x \to b}{\text{Lt}} f(x) = l$.
- If $\lim_{x \to a} f(x) = 0$, then the following results would be holding true.
  - $\lim_{x \to a} \frac{\sin f(x)}{f(x)} = \lim_{x \to a} \frac{\tan f(x)}{f(x)}$
    $= \lim_{x \to a} \cos f(x) = 1$
  - $\lim_{x \to a} \frac{\sin^{-1} f(x)}{f(x)} = \lim_{x \to a} \frac{\tan^{-1} f(x)}{f(x)} = 1$
  - $\lim_{x \to a} \frac{b^{f(x)} - 1}{f(x)} = \ln b \ (b > 0)$

Continuity

A function *f*(*x*) is said to be continuous if its graph has no break or hole on it.

Continuity of a Function at a Point

The real valued function $y = f(x)$ is said to be continuous at $x = a$ if $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$.

Continuity Everywhere

A function is said to be continuous everywhere if it is continuous on the entire real number line $(-\infty, \infty)$.

Continuity of a Function in an Interval

$f(x)$ is said to be continuous in an open interval (a, b) if it is continuous at each point of the interval.

$f(x)$ is said to be continuous in the closed interval $[a, b]$ if $f(x)$ is continuous in (a, b), then

$\lim_{x \to a^+} f(x) = f(a)$ and

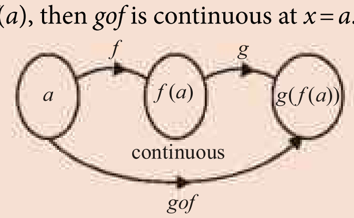
$\lim_{x \to b^-} f(x) = f(b)$.

Properties of Continuous Functions

- Let $f(x)$ and $g(x)$ be two continuous functions on their common domain D and α be any real number, then $\alpha f(x)$ is continuous ; $f \pm g$ is continuous ; fg is continuous; f/g is continuous, provided $g(x) \neq 0$ for any $x \in D$; $f^n(x), \forall \ n \in \mathbb{N}$ is continuous.
- The largest (greatest) integer function $[x]$ is continuous at all points except at integer points.

Continuity of Composition of Two Functions

If f, g are continuous functions then gof are continuous, if f is continuous at a point $x = a$ and g is continuous at $f(a)$, then gof is continuous at $x = a$.



Discontinuity

The function $f(x)$ will be discontinuous at $x = a$ if it is not continuous at that point.

Discontinuity of First Kind (Jump Discontinuity)

The function $f(x)$ is said to has discontinuity of first kind or jump discontinuity at $x = a$ if $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ both exists finitely but are not equal.

Discontinuity of Second Kind (Oscillating Discontinuity)

The function $f(x)$ is said to has discontinuity of second kind (oscillating discontinuity) at $x = a$ if $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ both does not exist finitely or do not approach a definite value of $x = a$.

Removable Discontinuity

The function $f(x)$ is said to has removable discontinuity at $x = a$ if $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ both exists and are equal but not equal to $f(a)$.

Differentiability

A differentiable function is a function in one variable such that its derivative exists at each point in its entire domain.

Definition

A real valued function $f(x)$ is differentiable at $x = a$ if $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$ exists finitely and it is denoted by $f'(a)$. In other words, $f(x)$ is said to be differentiable at $x = a$ if its left hand derivative *i.e.*, $Lf'(a)=\lim_{h\to 0}\frac{f(a-h)-f(a)}{-h}$ and right hand derivative *i.e.*, $Rf'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ at $x = a$ are exist and equal.

Note : Every differentiable function is continuous but the converse is not necessarily true.

Derivative of Functions

Composite Function (Chain Rule)

Let $y = f(t)$ and $t = g(x)$, then

$$\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}$$

Implicit Function

Here, we differentiate the function of type $f(x, y) = 0$, where x and y can't be expressed in terms of one another.

Parametric Function

If $x = f(t)$ and $y = g(t)$, then

$$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{g'(t)}{f'(t)},\ f'(t)\neq 0.$$

Logarithmic Differentiation

If $y = u^v$, where u, v are functions of x, then

$$\left[\log y = v\log u \Rightarrow \frac{d}{dx}(u^v) = u^v\left[\frac{v}{u}\frac{du}{dx}+\log u\frac{dv}{dx}\right]\right].$$

Basic Rules of Differentiation

Scalar Product: $\frac{d}{dx}(cf(x)) = c\frac{d}{dx}f(x) = cf'(x)\ \forall\ c \in R.$

Sum & Difference Rule: $\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x)) = f'(x) \pm g'(x)$

Product Rule: $\frac{d}{dx}[f(x) \cdot g(x)] = f'(x) \cdot g(x) + g'(x) \cdot f(x)$

Quotient Rule: $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$, provided $g(x)$, $f(x)$ both are differentiable functions and $g(x) \neq 0$.

Some Standard Derivatives

- $\frac{dk}{dx} = 0$, k is a constant
- $\frac{d}{dx}\sin x = \cos x$
- $\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x$
- $\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}\cot^{-1}x = \frac{-1}{1+x^2}$
- $\frac{d}{dx}e^x = e^x$
- $\frac{d}{dx}\ln\left(x+\sqrt{x^2-1}\right) = \frac{1}{\sqrt{x^2-1}}$
- $\frac{d}{dx}x^n = nx^{n-1}$, n is a constant
- $\frac{d}{dx}\cos x = -\sin x$
- $\frac{d}{dx}\sec x = \sec x \tan x$
- $\frac{d}{dx}\cos^{-1}x = \frac{-1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}\sec^{-1}x = \frac{1}{|x|\sqrt{x^2-1}}$
- $\frac{d}{dx}(a^x) = a^x \ln a$, $a > 0$, $a \neq 1$
- $\frac{d}{dx}\ln\left(\frac{1+x}{1-x}\right) = \frac{2}{1-x^2}$, $|x| < 1$
- $\frac{d}{dx}\ln x = \frac{1}{x}$, $x > 0$
- $\frac{d}{dx}\tan x = \sec^2 x$
- $\frac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x \cot x$
- $\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$
- $\frac{d}{dx}\operatorname{cosec}^{-1}x = \frac{-1}{|x|\sqrt{x^2-1}}$
- $\frac{d}{dx}\ln(x+\sqrt{x^2+1}) = \frac{1}{\sqrt{x^2+1}}$
- $\frac{d}{dx}x^n|x| = (n+1)x^{n-1}|x|$, $n \in N$

Application of Derivatives

Rate of Change of Quantities

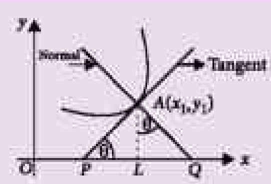
- Let $y = f(x)$, then $\frac{dy}{dx}$ or $f'(x)$ denotes the rate of change of y w.r.t. x.
- Velocity is the rate of change of displacement with respect to time *i.e.*, velocity $= \frac{d\vec{s}}{dt}$.
- Acceleration is the rate of change of velocity with respect to time *i.e.*, $\frac{dv}{dt} = \frac{d^2\vec{s}}{dt^2}$.

Errors and Approximations

Let $y = f(x)$, Δx be the small change in x and Δy be the corresponding change in y. Then, $\Delta y = \frac{dy}{dx}(\Delta x)$

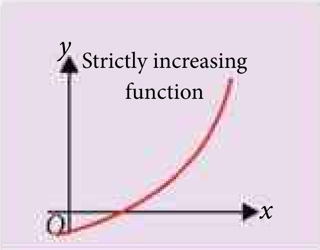
- **Absolute Error :** Δx
- **Relative Error :** $\Delta x/x$
- **Percentage Error :** $\left(\frac{\Delta x}{x} \times 100\right)$

Length of Tangents and Normals

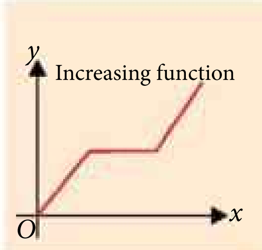


- Length of tangent $= AP = \left|\frac{y_1\sqrt{1+\left(\frac{dy_1}{dx_1}\right)^2}}{\frac{dy_1}{dx_1}}\right|$
- Length of normal $= AQ = \left|y_1\sqrt{1+\left(\frac{dy_1}{dx_1}\right)^2}\right|$
- Length of sub-tangent $= \left|\frac{y_1}{\frac{dy_1}{dx_1}}\right|$
- Length of sub-normal $= \left|y_1\frac{dy_1}{dx_1}\right|$

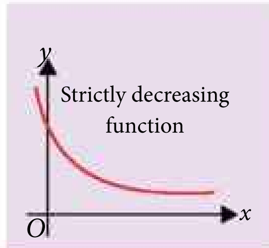
Increasing and Decreasing Functions



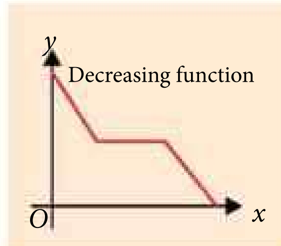
Strictly increasing on I if $x_1 < x_2$ in I $\Rightarrow f(x_1) < f(x_2)$ for all $x_1, x_2 \in I$



Increasing on I if $x_1 < x_2$ in I $\Rightarrow f(x_1) \leq f(x_2)$ for all $x_1, x_2 \in I$



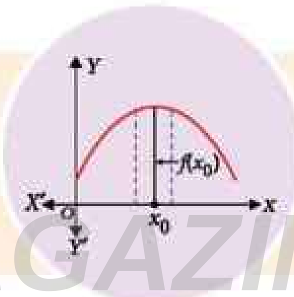
Strictly decreasing on I if $x_1 < x_2$ in I $\Rightarrow f(x_1) > f(x_2)$ for all $x_1, x_2 \in I$



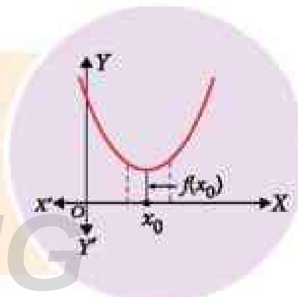
Decreasing on I if $x_1 < x_2$ in I $\Rightarrow f(x_1) \geq f(x_2)$ for all $x_1, x_2 \in I$

Maxima and Minima

Maximum value of a function : $f(x)$ is said to be maxima in I, if there exists a point $x_0 \in I$ such that $f(x_0) > f(x)\ \forall\ x \in I$.



Minimum Value of a Function : $f(x)$ is said to be minima in I, if there exists a point $x_0 \in I$ such that $f(x_0) < f(x)\ \forall\ x \in I$.



Rolle's Theorem

If $f(x)$ is a real valued function defined on $[a, b]$ is continuous on $[a, b]$ and differentiable on (a, b) such that $f(a) = f(b)$, then $\exists$ atleast one $c \in (a, b)$ such that $f'(c) = 0$.

Lagrange's Mean Value Theorem

If $f(x)$ is a real valued function defined on $[a, b]$ is continuous on $[a, b]$ and differentiable on (a, b), then $\exists$ atleast one $c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Differential Equations

An equation with one or more derivative of a function.

Solution of a Differential Equation

A function of the form $y = f(x) + c$, which satisfies the given differential equation, it can be a **General Solution,** which contains as many arbitrary constants as order of differential equation. **Particular Solution,** which is obtained by assigning values to arbitrary constants.

Methods of Solving the First Order First Degree Differential Equation

Variable Separable Form

If $\frac{dy}{dx} = f(x)g(y)$, then $\int \frac{dy}{g(y)} = \int f(x)\,dx + c$

Reducible to Variable Separate Form

If $dy/dx = f(ax + by + c)$, then put $ax + by + c = z$ and $b\frac{dy}{dx} = \frac{dz}{dx} - a$ and then apply variable separable method.

Homogeneous Differential Equation

If $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$, where $f(x, y)$, $g(x, y)$ are homogeneous functions of the same degree in x and y, then put $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ then apply variable separable methods.

Linear Differential Equation

If $\frac{dy}{dx} + Py = Q$, where P, Q are constants or functions of x, then $ye^{\int Pdx} = \int Qe^{\int Pdx}dx + c$, where $e^{\int Pdx}$ is the integrating factor (I.F.).

OR

If $\frac{dx}{dy} + Px = Q$, where P, Q are constants or functions of y, then $xe^{\int Pdy} = \int Qe^{\int Pdy}dy + c$, where $e^{\int Pdy}$ is the integrating factor (I.F.).

Some General Derivatives

- $d(x \pm y) = dx \pm dy$
- $d(xy) = xdy + ydx$
- $d\left(\frac{x}{y}\right) = \frac{ydx - xdy}{y^2}$
- $d\left(\frac{y}{x}\right) = \frac{xdy - ydx}{x^2}$
- $d(\log(xy)) = \frac{xdy + ydx}{xy}$
  $= \frac{dy}{y} + \frac{dx}{x}$
- $d(\log(x+y)) = \frac{dx+dy}{x+y}$
- $d\left(\log\left(\frac{x}{y}\right)\right) = \frac{ydx - xdy}{xy} = \frac{dx}{x} - \frac{dy}{y}$
- $d\left(\frac{1}{2}\log\left(\frac{x+y}{x-y}\right)\right) = \frac{xdy - ydx}{x^2 - y^2}$
- $d\left(\frac{1}{2}\log\left(\frac{x-y}{x+y}\right)\right) = \frac{ydx - xdy}{x^2 - y^2}$
- $d\left(\tan^{-1}\left(\frac{y}{x}\right)\right) = \frac{xdy - ydx}{x^2 + y^2}$
- $d\left(\tan^{-1}\left(\frac{x}{y}\right)\right) = \frac{ydx - xdy}{x^2 + y^2}$
- $d\left(\sqrt{x^2 \pm y^2}\right) = \frac{xdx \pm ydy}{\sqrt{x^2 \pm y^2}}$
- $\frac{d[g(x,y)]^{1-n}}{1-n} = \frac{g'(x,y)}{[g(x,y)]^n}$
- $d(\sin^{-1}(y/x)) =$
  $\frac{1}{\sqrt{x^2 - y^2}}\left(\frac{xdy - ydx}{x}\right)$
- $\frac{1}{2}d(\log(x^2 \pm y^2)) = \frac{xdx \pm ydy}{x^2 \pm y^2}$

Indefinite Integrals

The integrals that can be calculated by the reverse process of differentiation and are referred to as the antiderivatives of functions.

Methods of Integration

Using Substitution

The given integral $\int f(x)\,dx$ can be transformed into another form by changing the independent variable x to t by substituting $x = g(t)$.

Using By Parts

If u and v are two differentiable functions of x, then

$$\int (uv)\,dx = \left[u \cdot \int v dx\right] - \int \left\{\frac{du}{dx} \cdot \int v dx\right\} dx.$$

In order to choose 1st function, we take the letter which comes first in the word ILATE.

I – Inverse Trigonometric Function,

L – Logarithmic Function, A – Algebraic Function,

T–Trigonometric Function, E–Exponential Function

Using Partial Fractions

- If $f(x)$ and $g(x)$ are two polynomials such that $\deg f(x) \geq \deg g(x)$, then we divide $f(x)$ by $g(x)$.

  $$\therefore \frac{f(x)}{g(x)} = \text{Quotient} + \frac{\text{Remainder}}{g(x)}$$

- If $f(x)$ and $g(x)$ are two polynomials such that the degree of $f(x)$ is less than the degree of $g(x)$, then we can evaluate $\int \frac{f(x)}{g(x)} dx$ by decomposing $\frac{f(x)}{g(x)}$ into partial fraction.

Some Standard Integrals

- $\int dx = x + c$
- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$, where $n \neq -1$
- $\int e^x dx = e^x + c$
- $\int a^x dx = \frac{a^x}{\log_e a} + c$, where $a > 0$, $a \neq 1$
- $\int \frac{1}{x} dx = \log_e |x| + c$, where $x \neq 0$
- $\int \sin x\, dx = -\cos x + c$
- $\int \cos x\, dx = \sin x + c$
- $\int \tan x\, dx = \log|\sec x| + c$
- $\int \cot x\, dx = \log|\sin x| + c$
- $\int \sec x\, dx = \log|\sec x + \tan x| + c$
- $\int \text{cosec}\, x\, dx = \log|\text{cosec}\, x - \cot x| + c$

where 'c' is the constant of integration.

Integrals of Some Particular Functions

- $\int \frac{1}{\sqrt{x^2-a^2}}dx = \log\left|x+\sqrt{x^2-a^2}\right| + c$
- $\int \frac{1}{\sqrt{x^2+a^2}}dx = \log\left|x+\sqrt{x^2+a^2}\right| + c$
- $\int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$
- $\int \frac{1}{a^2-x^2}dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c$
- $\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$
- $\int \frac{1}{x\sqrt{x^2-1}}dx = \sec^{-1} x + c = -\operatorname{cosec}^{-1}x + c,$

  where $|x| > 1$
- $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + c$
- $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + c$
- $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$
- $\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
- $\int e^x[f(x) + f'(x)]dx = e^x f(x) + c$

Definite Integrals

When the limits are defined to generate a unique value, Definite Integrals are used.

Properties of Definite Integrals

- $\int_a^b f(x)dx = \int_a^b f(t)dt$
- $\int_a^b f(x)dx = -\int_b^a f(x)dx$
- $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx,$

  where $a < c < b$
- $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$
- $\int_0^a f(x)dx = \int_0^a f(a-x)dx$
- $\int_{-a}^a f(x)dx = \begin{cases} 0 & ,\text{if } f(-x) = -f(x) \\ 2\int_0^a f(x)dx & ,\text{if } f(-x) = f(x) \end{cases}$
- $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$
- $\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(2a-x) = f(x) \\ 0, & \text{if } f(2a-x) = -f(x) \end{cases}$
- $\int_\alpha^\beta f(x)dx = (\beta-\alpha)\int_0^1 f(\alpha+(\beta-\alpha)x)dx$
- Let $f(x)$ be periodic with period T. Then $\int_a^{a+nT} f(x)dx = n\int_0^T f(x)dx,\ n \in N$

JEE WORK OUTS

Single Option Correct Type

1. Suppose $f(x) = (x + 1)^2$ for $x \geq -1$. If $g(x)$ is the function whose graph is the reflection of the graph of $f(x)$ with respect to the line $y = x$, then $g(x)$ equals

(a) $-\sqrt{x} - 1, x \geq 0$ (b) $\frac{1}{(x+1)^2}, x > -1$

(c) $\sqrt{x+1}, x \geq -1$ (d) $\sqrt{x} - 1, x \geq 0$

2. If matrix A is circulant matrix whose elements of first row are a, b, c all > 0 such that $abc = 1$ and $A \cdot A^T = I$, then $a^3 + b^3 + c^3$ equals

(a) 0 (b) 4 (c) 1 (d) 3

3. The sum of the squares of the length of the chord intercepted by the line $x + y = n, n \in N$ on the circle $x^2 + y^2 = 16$ is

(a) 210 units (b) 180 units

(c) 150 units (d) 120 units

4. A coin is tossed $(2n + 1)$ times, the probability that head appear odd number of times is

(a) $\frac{n}{2n+1}$ (b) $\frac{n+1}{2n+1}$

(c) 1/2 (d) None of these

5. Let $S(k) = 1 + 3 + 5 + ... + (2k - 1) = 3 + k^2$.
Then which of the following is true?

(a) $S(k) \Rightarrow S(k - 1)$

(b) $S(k) \Rightarrow S(k + 1)$

(c) $S(1)$ is correct

(d) Principle of mathematical induction can be used to prove the formula.

6. If $2f(x) + xf\left(\frac{1}{x}\right) - 2f\left[\left|\sqrt{2}\sin\pi\left(x + \frac{1}{4}\right)\right|\right]$ $= 4\cos^2\left(\frac{\pi x}{2}\right) + x\cos\frac{\pi}{x}\ \forall\, x \in R - \{0\}$, then which of the following is false?

(a) $f(2) + f\left(\frac{1}{2}\right) = 1$ (b) $f(2) + f(1) = f\left(\frac{1}{2}\right)$

(c) $f(2) + f(1) = 0$ (d) $f(1)f\left(\frac{1}{2}\right)f(2) = 1$

7. If $\int_1^2 \frac{dx}{(x^2 - 2x + 4)^{3/2}} = \frac{k}{k+5}$, then k is equal to

(a) 1 (b) 3 (c) 4 (d) 2

8. Let a vector $\alpha\hat{i} + \beta\hat{j}$ be obtained by rotating the vector $\sqrt{3}\,\hat{i} + \hat{j}$ by an angle 45° about the origin in counterclockwise direction in the first quadrant. Then the area of triangle having vertices (α, β), $(0, \beta)$ and $(0, 0)$ is equal to

(a) 1 (b) $2\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{2}$

9. A triangle is inscribed in hyperbola $xy = a^2$ and two of its sides are parallel to the lines whose slopes are m_1 & m_2 and passing through origin. If m_1 & m_2 are roots of the equation $x^2 - 8x + 1 = 0$ and if the remaining side envelopes the hyperbola $xy = \alpha a^2$, then the value of α equals

(a) 4 (b) 6 (c) 8 (d) 16

10. The domain of the function

$f(x) = \sin^{-1}\left(\frac{|x| + 5}{x^2 + 1}\right)$ is $(-\infty, -a] \cup [a, \infty)$.

Then a is equal to

(a) $\frac{\sqrt{17}}{2}$ (b) $\frac{\sqrt{17} - 1}{2}$

(c) $\frac{1 + \sqrt{17}}{2}$ (d) $\frac{\sqrt{17}}{2} + 1$

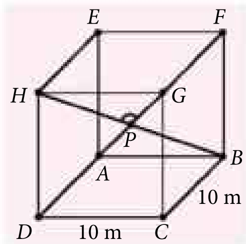
11. Let $f(x-y) = f(x)\,g(y) - f(y)\,g(x)$ and $g(x-y) = g(x)\,g(y) + f(x)\,f(y)$, $x, y \in R$. If right hand derivative of $f(x)$ exists at $x = 0$, then $g'(0) =$

(a) 0 (b) 1
(c) – 1 (d) none of these

12. $p \leftrightarrow q$ is equivalent to

(a) $(p \wedge q) \vee (\sim p \wedge \sim q)$ (b) $(p \wedge q) \vee (\sim p \wedge q)$
(c) $(p \vee q) \vee (\sim p \vee \sim q)$ (d) $(p \wedge q) \vee (\sim p \vee q)$

13. A hall has a square floor of dimension 10 m × 10 m (see the figure) and vertical walls. If the angle GPH between the diagonals AG and BH is $\cos^{-1}1/5$, then the height of the hall (in metres) is



(a) $5\sqrt{2}$ (b) $5\sqrt{3}$ (c) $2\sqrt{10}$ (d) 5

14. Let r be the range of n ($\forall\ n \geq 1$) observations $x_1, x_2, ..., x_n$, if $S = \sqrt{\dfrac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$, then

(a) $S < r\sqrt{\dfrac{n^2+1}{n-1}}$ (b) $S \geq r\sqrt{\dfrac{n}{n-1}}$

(c) $S = r\sqrt{\dfrac{n}{n-1}}$ (d) $S < r\sqrt{\dfrac{n}{n-1}}$

One or More Than One Option(s) Correct Type

15. Let S be the set of all non-zero real numbers α such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots x_1 and x_2 satisfying the inequality $|x_1 - x_2| < 1$. Which of the following intervals is(are) a subset(s) of S?

(a) $\left(-\dfrac{1}{2}, -\dfrac{1}{\sqrt{5}}\right)$ (b) $\left(-\dfrac{1}{\sqrt{5}}, 0\right)$

(c) $\left(0, \dfrac{1}{\sqrt{5}}\right)$ (d) $\left(\dfrac{1}{\sqrt{5}}, \dfrac{1}{2}\right)$

16. If $b + c$, $c + a$, $a + b$ are in H.P., then

(a) $\dfrac{b+c}{a}, \dfrac{c+a}{b}, \dfrac{a+b}{c}$ are in A.P.

(b) $\dfrac{b+c}{a}, \dfrac{c+a}{b}, \dfrac{a+b}{c}$ are in H.P.

(c) a^2, b^2, c^2 are in A.P.

(d) a^2, b^2, c^2 are in H.P.

17. For a given triangle whose vertices are (–12, 0), (0, 12) & (–14, 14), which of the following is true?

(a) Centre of incircle is (–9, 9).
(b) Radius of incircle is $3\sqrt{2}$.
(c) Equation of incircle is
$$(x+9)^2 + (y-9)^2 = \left(3\sqrt{2}\right)^2.$$
(d) Radius of incircle is $\dfrac{3}{\sqrt{2}}$.

18. For every twice differentiable function $f: R \to [-2, 2]$ with $(f(0))^2 + (f'(0))^2 = 85$, which of the following statement(s) is (are) TRUE?

(a) There exist $r, s \in R$, where $r < s$, such that f is one-one on the open interval (r, s)
(b) There exists $x_0 \in (-4, 0)$ such that $|f'(x_0)| \leq 1$
(c) $\lim\limits_{x\to\infty} f(x) = 1$
(d) There exists $\alpha \in (-4, 4)$ such that $f(\alpha) + f''(\alpha) = 0$ and $f'(\alpha) \neq 0$

19. Let $C_k = {}^nC_k$ for $0 \leq k \leq n$ and $A_k = \begin{pmatrix} C_{k-1}^2 & 0 \\ 0 & C_k^2 \end{pmatrix}$ for $k \geq 1$, and $A_1 + A_2 + ... + A_n = \begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix}$, then

(a) $k_1 = k_2$ (b) $k_1 + k_2 = {}^{2n}C_{2n} + 1$
(c) $k_1 = {}^{2n}C_n - 1$ (d) $k_2 = {}^{2n}C_{n+1}$

20. The point in which the line $\dfrac{x-1}{1} = \dfrac{y-1}{-1} = \dfrac{z+3}{1}$ cuts the surface $x^2 + y^2 + z^2 - 20 = 0$ is

(a) (0, 2, 4) (b) (0, 2, – 4)
(c) (4, –2, 0) (d) (0, – 2, – 4)

21. Let $f : R \to R$ and $g : R \to R$ be two non-constant differentiable functions. If $f'(x) = (e^{(f(x) - g(x))})g'(x)$ for all $x \in R$ and $f(1) = g(2) = 1$, then which of the following statement(s) is (are) TRUE?

(a) $f(2) < 1 - \log_e 2$ (b) $f(2) > 1 - \log_e 2$
(c) $g(1) > 1 - \log_e 2$ (d) $g(1) < 1 - \log_e 2$

22. Let $S_1 = \{(i, j, k) ; i, j, k \in \{1, 2, ..., 10\}\}$,
$S_2 = \{(i, j) : 1 \leq i < j + 2 \leq 10; i, j \in \{1, 2, ..., 10\}\}$,
$S_3 = \{(i, j, k, l) : 1 \leq i < j < k < l; i, j, k, l \in \{1, 2, ..., 10\}\}$
and $S_4 = \{(i, j, k, l) ; i, j, k$ and l are distinct elements in $\{1, 2, ..., 10\}\}$.
If the total number of elements in the set S_r is n_r, $r = 1, 2, 3, 4$, then which of the following statements is (are) TRUE?

(a) $n_1 = 1000$ (b) $n_2 = 44$
(c) $n_3 = 220$ (d) $\dfrac{n_4}{12} = 420$

23. Let $f(x)=\sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$ for all $x\in R$ and $g(x)=\frac{\pi}{2}\sin x$ for all $x\in R$. Let $(fog)(x)$ denote $f(g(x))$ and $(gof)(x)$ denote $g(f(x))$. Then which of the following is (are) true?

(a) Range of f is $\left[-\frac{1}{2},\frac{1}{2}\right]$.

(b) Range of fog is $\left[-\frac{1}{2},\frac{1}{2}\right]$.

(c) $\lim\limits_{x\to 0}\frac{f(x)}{g(x)}=\frac{\pi}{6}$

(d) There is an $x\in R$ such that $(gof)(x) = 1$.

Matrix Match Type

24. Match the following columns and choose the correct option.

Column - I		Column - II	
(P)	$\int_{-1}^{3}\frac{dx}{\sqrt{(x+1)(3-x)}}$	(1)	$\frac{\pi}{2}$
(Q)	$\frac{1}{8}\int_{-1}^{3}\sqrt{\frac{x+1}{3-x}}\,dx$	(2)	$\frac{\pi}{4}$
(R)	$\frac{1}{4}\int_{-1}^{3}\sqrt{\frac{3-x}{x+1}}\,dx$	(3)	π
(S)	$\int_{-1}^{3}\sqrt{(x+1)(3-x)}\,dx$	(4)	2π

	P	Q	R	S
(a)	1	2	3	4
(b)	3	1	2	4
(c)	2	3	4	1
(d)	3	2	1	4

25. Match the following columns and choose the correct option.

Column-I		Column-II	
(P)	If the area bounded by the smaller region of $\lvert x+y\rvert+\lvert x-y\rvert=2$ and $y=x^2$ is Δ, then the value of $3\Delta/2$ is	(1)	6
(Q)	Consider the area bounded by the x-axis and part of the curve $y=1+8/x^2$ between $x=2$ and $x=4$. If the ordinate at $x=a$ divides the area into two equal parts, then the value of a^2 is	(2)	4
(R)	If the area bounded by the polynomial $y=x^2-\lvert x^2-1\rvert+2\lvert\lvert x\rvert-1\rvert+2\lvert x\rvert-7$ and the x-axis is A, then the value of $\frac{3A}{11}$ is	(3)	2
(S)	If the area of the region bounded by the x-axis and the curves defined by $y=\tan x$, $\left(-\frac{\pi}{3}\le x\le\frac{\pi}{3}\right)$ and $y=\cot x$, $\left(\frac{\pi}{6}\le x\le\frac{3\pi}{2}\right)$ is $\log_e(9/k)$, then the value of k is	(4)	8

	P	Q	R	S
(a)	1	2	3	4
(b)	3	4	2	1
(c)	2	3	1	4
(d)	3	2	4	1

Numerical Answer Type

26. Let $a_1, a_2, a_3, \ldots\ldots\ldots, a_{11}$ be real numbers satisfying $a_1=15$, $27-2a_2>0$ and $a_k=2a_{k-1}-a_{k-2}$ for $\frac{\pi}{2}$ $k=3,4,\ldots,11$. If $\frac{a_1^2+a_2^2+\ldots\ldots+a_{11}^2}{11}=90$, then the value of $\frac{a_1+a_2+\ldots\ldots+a_{11}}{11}$ is ______ .

27. The number of integral values of a for which a unique circle passes through the points of intersection of the rectangular hyperbola $x^2-y^2=a^2$ and the parabola $y=x^2$ is ______ .

28. Let $(2x^2+3x+4)^{10}=\sum\limits_{r=0}^{20}a_r x^r$.

Then $\frac{a_7}{a_{13}}$ is equal to ______.

29. If $\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}}=\frac{1}{7}$ and $\sqrt{\frac{1-\cos 2\beta}{2}}=\frac{1}{\sqrt{10}}$, $\alpha,\beta\in\left(0,\frac{\pi}{2}\right)$, then $\tan(\alpha+2\beta)$ is equal to ______.

30. Let $f: R\to R$ be a differentiable function such that $f(0)=0$, $f\left(\frac{\pi}{2}\right)=3$ and $f'(0)=1$.

If $g(x)=\int_x^{\pi/2}[f'(t)\operatorname{cosec} t-\cot t \operatorname{cosec} t\ f(t)]\,dt$ for

$x\in\left(0,\dfrac{\pi}{2}\right]$, then $\lim_{x\to 0} g(x) =$ ______.

SOLUTIONS

1. (d): $f(x)=(x+1)^2, x\ge -1$

Now if $g(x)$ is the reflection of $f(x)$ in the line $y=x$, then it can be obtained by interchanging x and y in $f(x)$

i.e., $y=(x+1)^2$ changes to $x=(y+1)^2$

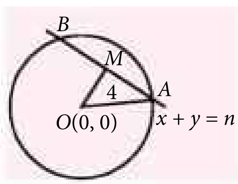
$\Rightarrow\ y+1=\sqrt{x}\ \Rightarrow\ y=\sqrt{x}-1$ defined $\forall x\ge 0$

$\therefore\ g(x)=\sqrt{x}-1, \forall x\ge 0.$

2. (b)

3. (a): Chord intercepted by the line $x+y=n$ is AB. Let M is mid point of the chord AB.

$\therefore\ BM=MA=\dfrac{1}{2}AB$



i.e., $AB=2BM=2MA$

According to question, we have

$(AB)^2=4(MA)^2$

$=4[(OA)^2-(OM)^2]=4\left[16-\left(\dfrac{n}{\sqrt{2}}\right)^2\right]$

($\because$ OM = distance from (0, 0) to the line $x+y=n$)

$=2[32-n^2], n\in N, 1\le n<6$

$=2[(32-1^2)+(32-2^2)+(32-3^2)+(32-4^2)+(32-5^2)]$

$=2[32\times 5-(1^2+2^2+3^2+4^2+5^2)]$

$=2\left[160-\dfrac{5\cdot 6\cdot 11}{6}\right]=2\times 105=210$ units

4. (c): $P(H)=P(T)=1/2$ (with a single coin)

The required probability = P(head occurs 1 or 3 ..... or $(2n+1)$ times)

$\therefore\ P(X=r)={}^{2n+1}C_r\left(\dfrac{1}{2}\right)^r\left(\dfrac{1}{2}\right)^{2n+1-r}$

$\therefore\ P(1)+P(3)+P(5)+\ldots+P(2n+1)$

$={}^{2n+1}C_1\left(\dfrac{1}{2}\right)^1\left(\dfrac{1}{2}\right)^{2n}+{}^{2n+1}C_3\left(\dfrac{1}{2}\right)^3\left(\dfrac{1}{2}\right)^{2n-2}+\ldots+{}^{2n+1}C_{2n+1}\left(\dfrac{1}{2}\right)^{2n+1}$

$=\dfrac{1}{2^{2n+1}}[{}^{2n+1}C_1+{}^{2n+1}C_3+\ldots+{}^{2n+1}C_{2n+1}]$

$=\dfrac{1}{2^{2n+1}}\times 2^{2n}=\dfrac{1}{2}$

5. (b)

6. (d): $2f(x)+xf\left(\dfrac{1}{x}\right)-2f\left[\left|\sqrt{2}\sin\pi\left(x+\dfrac{1}{4}\right)\right|\right]$

$=4\cos^2\left(\dfrac{\pi x}{2}\right)+x\cos\dfrac{\pi}{x}$ ...(i)

When $x=2$, we have

$2f(2)+2f\left(\dfrac{1}{2}\right)-2f\left[\left|\sqrt{2}\sin\dfrac{\pi}{4}\right|\right]=4\cos^2(\pi)+2\cos\dfrac{\pi}{2}$

$\Rightarrow\ 2f(2)+2f\left(\dfrac{1}{2}\right)-2f(1)=4+0$

$\Rightarrow\ f(2)+f\left(\dfrac{1}{2}\right)=f(1)+2$ ...(ii)

When $x=1$, we have

$2f(1)+f(1)-2f(|-1|)=-1\ \Rightarrow\ f(1)=-1$ ...(*)

From (ii), we get $f(2)+f\left(\dfrac{1}{2}\right)=1$ ...(iii)

When $x=\dfrac{1}{2}$ then from (i), we get

$2f\left(\dfrac{1}{2}\right)+\dfrac{1}{2}f(2)-2f(|1|)=4\left(\dfrac{1}{2}\right)+\dfrac{1}{2}$

$\Rightarrow\ 2f\left(\dfrac{1}{2}\right)+\dfrac{1}{2}f(2)+2=2+\dfrac{1}{2}$

$\Rightarrow\ 2f\left(\dfrac{1}{2}\right)+\dfrac{1}{2}f(2)=\dfrac{1}{2}$ ...(iv)

Solving (iii) & (iv), we get $f\left(\dfrac{1}{2}\right)=0$ and $f(2)=1$

as $f(2)=1\Rightarrow f(2)=-f(1)$ (using (*)) $\Rightarrow f(2)+f(1)=0$

7. (a): Let $I=\int_1^2\dfrac{dx}{(x^2-2x+4)^{3/2}}=\int_1^2\dfrac{dx}{((x-1)^2+3)^{3/2}}$

Put $x-1=\sqrt{3}\tan\theta\ \Rightarrow\ dx=\sqrt{3}\sec^2\theta d\theta$

When $x=1$, $\theta=0$ and when $x=2$, $\theta=\dfrac{\pi}{6}$

$\therefore\ I=\int_0^{\pi/6}\dfrac{\sqrt{3}\sec^2\theta d\theta}{(3\tan^2\theta+3)^{3/2}}$

$=\int_0^{\pi/6}\dfrac{\sqrt{3}\sec^2\theta\,d\theta}{3\sqrt{3}(\sec^2\theta)^{3/2}}=\int_0^{\pi/6}\dfrac{1}{3\sec\theta}d\theta$

$=\dfrac{1}{3}\int_0^{\pi/6}\cos\theta\,d\theta=\dfrac{1}{3}(\sin\theta)_0^{\pi/6}=\dfrac{1}{3}\left(\dfrac{1}{2}-0\right)=\dfrac{1}{6}$

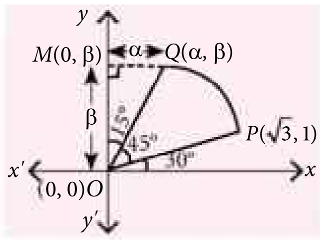
Now, $\dfrac{1}{6}=\dfrac{k}{k+5}\Rightarrow k+5=6k\ \Rightarrow\ 5k=5\ \Rightarrow\ k=1$

8. (d): Let $\overrightarrow{OP} = \sqrt{3}\,\hat{i} + \hat{j}$ and $\overrightarrow{OQ} = \alpha\,\hat{i} + \beta\,\hat{j}$

$\therefore\ |\overrightarrow{OP}| = |\overrightarrow{OQ}| = \sqrt{3+1} = 2$

In ΔOMQ, $\dfrac{\beta}{2} = \cos 15°$

and $\dfrac{\alpha}{2} = \sin 15°$ ...(i)



Area of (ΔOMQ)

$= \dfrac{1}{2} \times OM \times MQ$

$= \dfrac{1}{2}\alpha\beta = \dfrac{1}{2}(2\sin 15°)(2\cos 15°)$ [Using (i)]

$= 2\sin 15° \cos 15° = \sin 30° = \dfrac{1}{2}$

9. (d): Let ABC be a triangle inscribed in $xy = a^2$ therefore the co-ordinates of the vertices of the triangle be $A\left(at_1, \dfrac{a}{t_1}\right), B\left(at_2, \dfrac{a}{t_2}\right)$ & $C\left(at_3, \dfrac{a}{t_3}\right)$, t_1, t_2, t_3 are parameters.

The equation of line AB is

$x + yt_1t_2 = a(t_1 + t_2)$ ...(1)

and equation of the line BC is

$x + yt_2t_3 = a(t_2 + t_3)$ ...(2)

and equation of AC is $x + t_1t_3y = a(t_1 + t_3)$ ...(3)

Suppose lines (1) & (2) are respectively parallel to $y = m_1x$ & $y = m_2x$

$\therefore\ m_1 = -\dfrac{1}{t_1t_2}, m_2 = -\dfrac{1}{t_2t_3}$

$\Rightarrow\ \dfrac{m_1}{m_2} = \dfrac{t_3}{t_1} \Rightarrow t_3 = \left(\dfrac{m_1}{m_2}\right)t_1$ ...(4)

From (3) & (4) we have

$$x + y\left(\frac{m_1}{m_2}\right)t_1^2 = a\left(\frac{m_1}{m_2}t_1 + t_1\right)$$

$\Rightarrow\ ym_1t_1^2 - at_1(m_1 + m_2) + xm_2 = 0$ ...(5)

Now t_1 being a parameter, $t_1 \in (-\infty, \infty)$ is real envelope of equation (5)

$\therefore\ D = 0 \Rightarrow a^2(m_1 + m_2)^2 - 4xym_1m_2 = 0$

$\Rightarrow\ 4xym_1m_2 = a^2(m_1 + m_2)^2$ but m_1 & m_2 are roots of $x^2 - 8x + 1 = 0$

$\therefore\ m_1 + m_2 = 8$ & $m_1m_2 = 1$

Now, $4m_1m_2xy = a^2(m_1 + m_2)^2$

$\Rightarrow\ 4xy = a^2(8)^2 \Rightarrow xy = 16a^2 = \alpha a^2$ (given) $\Rightarrow\ \alpha = 16$

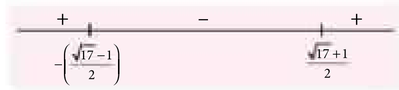
10. (c): Here, $f(x) = \sin^{-1}\left(\dfrac{|x|+5}{x^2+1}\right)$

$\Rightarrow\ \left|\dfrac{|x|+5}{x^2+1}\right| \le 1 \Rightarrow |x| + 5 \le x^2 + 1 \Rightarrow x^2 - 4 - |x| \ge 0$

$\Rightarrow\ \left(|x| - \dfrac{1}{2}\right)^2 - \dfrac{17}{4} \ge 0 \Rightarrow \left(|x| - \dfrac{1}{2}\right)^2 - \left(\dfrac{\sqrt{17}}{2}\right)^2 \ge 0$

$\Rightarrow\ \left(|x| - \dfrac{1}{2} + \dfrac{\sqrt{17}}{2}\right)\left(|x| - \dfrac{1}{2} - \dfrac{\sqrt{17}}{2}\right) \ge 0$

$\Rightarrow\ \left(|x| + \dfrac{\sqrt{17}-1}{2}\right)\left(|x| - \left(\dfrac{\sqrt{17}+1}{2}\right)\right) \ge 0$



But $|x|$ can't be negative, therefore $|x| \ge \dfrac{\sqrt{17}+1}{2}$

$\Rightarrow\ x \in \left(-\infty, -\left(\dfrac{\sqrt{17}+1}{2}\right)\right] \cup \left[\dfrac{\sqrt{17}+1}{2}, \infty\right)$

Thus $a = \dfrac{\sqrt{17}+1}{2}$

[$\because$ Domain of $f(x)$ is $(-\infty, -a] \cup [a, \infty)$]

SAMURAI SUDOKU

ANSWER - APRIL 2023

2	4	9	6	1	3	7	8	5				5	6	8	3	9	2	4	7	1
1	5	8	9	7	4	2	6	3				4	7	3	1	6	5	9	8	2
3	7	6	8	2	5	9	4	1				9	2	1	8	7	4	5	6	3
8	9	5	4	6	7	1	3	2				8	1	6	2	3	9	7	4	5
6	1	7	5	3	2	4	9	8				2	9	7	5	4	8	3	1	6
4	3	2	1	9	8	6	5	7				3	5	4	6	1	7	8	2	9
5	8	1	2	4	6	3	7	9	1	8	2	6	4	5	7	2	3	1	9	8
9	6	3	7	8	1	5	2	4	3	6	7	1	8	9	4	5	6	2	3	7
7	2	4	3	5	9	8	1	6	5	9	4	7	3	2	9	8	1	6	5	4
						4	9	8	7	2	1	5	6	3						
						6	5	2	4	3	8	9	1	7						
						1	3	7	9	5	6	4	2	8						
9	3	6	5	7	8	2	4	1	8	7	5	3	9	6	2	8	7	1	5	4
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1	4	3	7	9	2	8	5	6				4	6	8	1	7	2	5	3	9
2	9	5	4	8	6	1	3	7				7	3	9	6	5	4	2	8	1
7	6	8	1	5	3	4	2	9				1	2	5	8	3	9	4	7	6
3	5	2	8	1	7	6	9	4				5	4	3	7	1	8	9	6	2
6	1	9	2	3	4	5	7	8				6	1	7	9	2	5	3	4	8
4	8	7	9	6	5	3	1	2				9	8	2	3	4	6	7	1	5

11. (a) : $f(0) = 0$ by setting $x = y$ in the first relation.
$g(0) = 1$ by setting $y = 0$ in the first relation.
$f(-y) = -f(y)$ by setting $x = 0$ in the first relation.
$f(x)$ is an odd function and $f'(x)$ is an even function.
Hence left derivative = right derivative.
$\therefore f'(0)$ exists
$1 = g(0) = f^2(x) + g^2(x)$ by setting $x = y$ in the second relation.
Differentiating, $f^2(x) + g^2(x) = 1$, we get
$2f(x)f'(x) + 2g'(x)g(x) = 0$
$x = 0 \Rightarrow g'(0) = 0$ since $f(0) = 0$, $g(0) = 1$.

12. (a) : $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$
$\equiv (\sim p \vee q) \wedge (\sim q \vee p)$
$\equiv [\sim p \wedge (\sim q \vee p)] \vee [q \wedge (\sim q \vee p)]$
$\equiv [(\sim p \wedge \sim q) \vee (\sim p \wedge p)] \vee [(q \wedge \sim q) \vee (q \wedge p)]$
$\equiv [(\sim p \wedge \sim q) \vee \text{F}] \vee [\text{F} \vee (q \wedge p)]$
$\equiv (\sim p \wedge \sim q) \vee (q \wedge p) \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

13. (a) : Let A is the origin, AD is along x axis, AB is along y axis, AE is along z axis. Then,
$A \equiv (0, 0, 0)$; $G \equiv (10, 10, h)$; $H \equiv (10, 0, h)$; $B \equiv (0, 10, 0)$
$\therefore \quad \overrightarrow{AG} = 10\hat{i} + 10\hat{j} + h\hat{k}$ and $\overrightarrow{BH} = 10\hat{i} - 10\hat{j} + h\hat{k}$

Also, we have $\cos\theta = \frac{1}{5} = \left|\frac{\overrightarrow{AG}\cdot\overrightarrow{BH}}{|\overrightarrow{AG}|\cdot|\overrightarrow{BH}|}\right|$, where θ is the angle between the diagonals AG and BH.

$$\Rightarrow \quad \frac{100 - 100 + h^2}{(100 + 100 + h^2)} = \frac{h^2}{200 + h^2} = \frac{1}{5}$$

$$\Rightarrow \quad 5h^2 = 200 + h^2 \Rightarrow h^2 = 50 \Rightarrow h = 5\sqrt{2}$$

14. (d): Here range = r = largest value – smallest value

$= \text{Max } |(x_i - x_j)|\ (i \neq j)$ and $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$

Now, consider $(x_i - \bar{x})^2 = \left[x_i - \frac{x_1 + x_2 + \ldots.. + x_n}{n}\right]^2$

$$= \frac{1}{n^2}\ [(x_i - x_1) + (x_i - x_2) + \ldots + (x_i - x_n)]^2$$

$$= \frac{1}{n^2}\ [(x_i - x_1) + (x_i - x_2) + \ldots$$
$$\ldots + (x_i - x_{i-1}) + (x_i - x_{i+1}) + \ldots + (x_i - x_n)]^2$$

$$\Rightarrow \quad (x_i - \bar{x})^2 \le \frac{1}{n^2}[(n-1)r]^2 \qquad \left(\because \left|x_i - x_j\right| \le r\right)$$

$$\Rightarrow \quad \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1} \le \frac{1}{n^2(n-1)}\sum_{i=1}^{n}[(n-1)r]^2$$

$$= \frac{1}{n^2}\frac{1}{n-1}n(n-1)^2r^2$$

$$= \frac{n-1}{n}r^2 < \frac{n}{n-1}\cdot r^2 \qquad \left(\because \forall n > 1, n > \frac{1}{n}\right)$$

$$\therefore \quad S^2 < \frac{n}{n-1}\cdot r^2 \text{ or } S < r\sqrt{\frac{n}{n-1}}$$

15. (a, d) : We have $\alpha x^2 - x + \alpha = 0$, $D = 1 - 4\alpha^2$

For distinct real roots, $1 - 4\alpha^2 > 0$ *i.e.*, $\alpha \in \left(-\frac{1}{2}, \frac{1}{2}\right)$

Now, $|x_1 - x_2| < 1 \Rightarrow \sqrt{(x_1 + x_2)^2 - 4x_1x_2} < 1$

$$\Rightarrow \frac{1}{\alpha^2} - 4 < 1 \Rightarrow 1 - 4\alpha^2 < \alpha^2$$

$$\Rightarrow \quad \alpha \in \left(-\infty, -\frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$$

Combining the two bounds, we have

$$\alpha \in \left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$$

$$\therefore \quad S = \left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$$

16. (b, c) : $b + c$, $c + a$, $a + b$ are in H.P. ...(i)

$\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.

Multiplying by $a + b + c$,

$1 + \frac{a}{b+c}, 1 + \frac{b}{c+a}, 1 + \frac{c}{a+b}$ are in A.P.

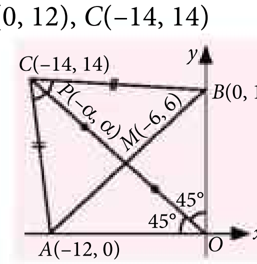
$\therefore \quad \frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in A.P.

$\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in H.P.

Multiply (i) by $(a + b)(b + c)(c + a)$ to get
$(c + a)(a + b)$, $(b + c)(a + b)$, $(b + c)(c + a)$ are in A.P.
$a^2 + \Sigma ab$, $b^2 + \Sigma ab$, $c^2 + \Sigma ab$ are in A.P.
$\therefore \quad a^2, b^2, c^2$ are in A.P.

17. (a, b, c) : Let $A(-12, 0)$, $B(0, 12)$, $C(-14, 14)$

$\therefore \quad AB = \sqrt{144 + 144}$
$= \sqrt{2 \cdot 144} = 12\sqrt{2}$
$BC = \sqrt{196 + 4}$
$= 10\sqrt{2} = AC$



$\therefore \quad \Delta ABC$ is an isosceles triangle.
Therefore the median through C is bisector of $\angle C$ and the equation of angle bisector of C passes through the mid point of AB where mid point of AB is $M(-6, 6)$.

Now, equation of angle bisector of C is OC or CM given by $y - 6 = -1(x + 6)$

$\Rightarrow$ $y = -x$, let any point on this line be $P(-\alpha, \alpha)$ where $\alpha > 0$.

Again, equation of AB is $x - y + 12 = 0$

and equation of AC is $7x + y + 84 = 0$

Now, the $\perp$ distance from $(-\alpha, \alpha)$ to the lines AB & AC is the radius and $(-\alpha, \alpha)$ be the centre of incircle.

Also, the $\perp$ distance from $(-\alpha, \alpha)$ to the lines AB & AC are equal.

$$\therefore \left|\frac{-7\alpha + \alpha + 84}{\sqrt{50}}\right| = \left|\frac{-\alpha - \alpha + 12}{\sqrt{2}}\right|$$

$$\Rightarrow \frac{-6\alpha + 84}{5} = \pm(-2\alpha + 12)$$

$\Rightarrow$ $-6\alpha + 84 = -5(-2\alpha + 12)$ and $-6\alpha + 84 = 5(-2\alpha + 12)$

$\Rightarrow$ $\alpha = 9$ and $\alpha = -6$ (rejected as $\alpha > 0$)

$\therefore$ Centre of incircle is $(-9, 9)$ and

$$\text{Radius} = \left|\frac{-2\alpha + 12}{\sqrt{2}}\right| = \frac{6}{\sqrt{2}} = 3\sqrt{2}$$

Thus, equation of incircle is $(x + 9)^2 + (y - 9)^2 = (3\sqrt{2})^2$

18. (a, b, d) : As f is continuous and it is not constant, we must have an interval (r, s) on which f is one-one.

Applying L.M.V.T. on $(-4, 0)$, we have

$$\frac{f(0) - f(-4)}{4} = f'(c), \text{where } c \in (-4, 0)$$

$$\Rightarrow \quad |f'(c)| \le \left|\frac{2 + 2}{4}\right| = 1$$

$\lim_{x\to\infty} f(x) = 1$ is incorrect as a counter example.

$f(x) = \sin(\sqrt{85}x)$ satisfies the given conditions but $\lim_{x\to\infty} \sin(\sqrt{85}x)$ doesn't exist.

Consider $A(x) = f^2(x) + f'^2(x);\ A(0) = 85$

$\Rightarrow$ $c \in (-4, 0)$ such that $|f'(x)|^2 \le 1$

$A(c) = f^2(c) + (f'(c))^2 \le 4 + 1 = 5$, then $c \in (-4, 0)$

Similarly, let d be such that $A(d) \le 5$, then $d \in (0, 4)$

$A(0) = 85$

$\therefore$ $A(x)$ must have maxima in (c, d), say at β

$\Rightarrow$ $A'(\beta) = 0$ and $A(\beta) \ge 85$

Now, $2f'(\beta)\ [f(\beta) + f''(\beta)] = 0$

$f'(\beta) = 0$ is not possible, hence $f(\beta) + f''(\beta) = 0$.

19. (a, c) : $A_1 = \begin{bmatrix} C_0^2 & 0 \\ 0 & C_1^2 \end{bmatrix};\ A_2 = \begin{bmatrix} C_1^2 & 0 \\ 0 & C_2^2 \end{bmatrix}; \ldots ;\ A_n = \begin{bmatrix} C_{n-1}^2 & 0 \\ 0 & C_n^2 \end{bmatrix}$

Now, $A_1 + A_2 + ... + A_n$

$$= \begin{bmatrix} C_0^2 + C_1^2 + ... + C_{n-1}^2 & 0 \\ 0 & C_1^2 + C_2^2 + ... + C_n^2 \end{bmatrix}$$

$$= \begin{bmatrix} C_0^2 + C_1^2 + ... + C_n^2 - C_n^2 & 0 \\ 0 & C_0^2 + C_1^2 + ... + C_n^2 - C_0^2 \end{bmatrix}$$

$$= \begin{bmatrix} {}^{2n}C_n - 1 & 0 \\ 0 & {}^{2n}C_n - 1 \end{bmatrix}$$

So, $k_1 = k_2 = {}^{2n}C_n - 1$

20. (b, c) : Given, $\dfrac{x-1}{1} = \dfrac{y-1}{-1} = \dfrac{z+3}{1} = \lambda$ (say)

QS World University Rankings 2023: IIT Guwahati among world's best for 14 Subjects

The 2023 edition of the QS World University Rankings by Subject, released by QS Quacquarelli Symonds, named the world's best universities for the study of 54 academic disciplines. Indian Institute of Technology (IIT) Guwahati has secured a place among the world's top universities for the study of 14 subjects, according to the latest edition of the world's most-consulted university ranking.

The 2023 edition of the QS World University Rankings by Subject, released by QS Quacquarelli Symonds, named the world's best universities for the study of 54 academic disciplines. IIT Guwahati has been ranked as one of the world's top universities in Petroleum Engineering, in which it ranks 51-100 globally and second in India.

Compared to the previous year, the institution is ranked in two additional subjects. For IIT Guwahati, six of its programs improved in rank. The institute has ranked between 201-250 in chemical engineering, mechanical, aeronautical and manufacturing engineering, electrical and electronics engineering, and 222nd best engineering and technology discipline in the world.

According to reports, IIT Guwahati has performed remarkably in the QS World University Rankings by Subject 2023. Prof Parameswar K Iyer, Officiating Director, IIT Guwahati, said, "IIT Guwahati is working hard to deliver quality education, which will play an important role in building a brighter future for all. Aligning with the National Education Policy 2020, the Institute is further improving research and development collaboration in multidisciplinary subjects. This has been possible through the collective efforts by all the faculty and students." Furthermore, IIT Guwahati performed well in Employer Reputation, where it scored 67.5 in Mathematics.

This year, three new academic disciplines have been included in the QS World University Rankings by Subject: Data Science, Marketing, and History of Art. The QS World University Rankings by Subject uses four metrics to rank universities: academic reputation, employer reputation, research citations per paper and H-index.

$\therefore \quad x = \lambda + 1, y = -\lambda + 1, z = \lambda - 3$

Now, $x^2 + y^2 + z^2 - 20 = 0$

$\Rightarrow \quad (\lambda + 1)^2 + (-\lambda + 1)^2 + (\lambda - 3)^2 - 20 = 0$

$\Rightarrow \quad 3\lambda^2 - 6\lambda + 11 - 20 = 0 \quad \Rightarrow \quad 3(\lambda^2 - 2\lambda - 3) = 0$

$\Rightarrow \quad \lambda^2 - 2\lambda - 3 = 0 \quad \Rightarrow \quad \lambda = -1, 3$

$\therefore$ Required points are (0, 2, – 4) and (4, – 2, 0).

21. (b, c) : We have $f'(x) = e^{(f(x)-g(x))} g'(x)$

$\Rightarrow \quad e^{-f(x)} f'(x) = e^{-g(x)} g'(x)$

Integrating, we get $-e^{-f(x)} = -e^{-g(x)} + \lambda$

i.e., $e^{-g(x)} - e^{-f(x)} = \lambda$

Putting $x = 1$ and $x = 2$ respectively, we get

$e^{-g(1)} - e^{-f(1)} = \lambda$ and $e^{-g(2)} - e^{-f(2)} = \lambda$

We have, $e^{-g(1)} - e^{-f(1)} = e^{-g(2)} - e^{-f(2)}$

$\Rightarrow \quad e^{-g(1)} - e^{-1} = e^{-1} - e^{-f(2)} \Rightarrow e^{-g(1)} + e^{-f(2)} = 2/e$

$\Rightarrow \quad e^{-f(2)} < \frac{2}{e} \Rightarrow -f(2) < \log_e 2 - 1 \therefore f(2) > 1 - \log_e 2$

Similar things holds for $g(1)$.

22. (a, b, d) : $n_1 = 10 \cdot 10 \cdot 10 = 1000$, as each of i, j and k have 10 choices.

For S_2, we do casework on j.

Given, $i < j + 2$ *i.e.*, $i \le j + 1$

$j = 1 \quad ; \quad i = 1, 2 \quad \to$ 2 choices

$j = 2 \quad ; \quad i = 1, 2, 3 \quad \to$ 3 choices

...

$j = 8 \quad ; \quad i = 1, 2, 3, \ldots, 9 \quad \to$ 9 choices

Hence, $n_2 = 2 + 3 + .. + 9 = (1 + 2 + \ldots + 9) - 1$

$= \frac{9 \times 10}{2} - 1 = 45 - 1 = 44$

$n_3 = {}^{10}C_4 = \frac{10 \cdot 9 \cdot 8 \cdot 7}{24} = 210$

$n_4 = {}^{10}C_4 \cdot 4! = 210 \cdot 4!$

Hence, $\frac{n_4}{12} = 420$

23. (a, b, c) : We have $f(x) = \sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$

As $x \in R$

$\sin\left(\frac{\pi}{2}\sin x\right) \in [-1, 1]; \ \frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right) \in \left[-\frac{\pi}{6}, \frac{\pi}{6}\right]$

$f(x) \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

$fog(x) = \sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin\left(\frac{\pi}{2}\sin x\right)\right)\right)$

It is observed that as $\frac{\pi}{2}\sin x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, the range of $fog(x)$ is also $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

Again, $\lim_{x\to 0} \frac{\sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)}{\frac{\pi}{2}\sin x}$

$= \lim_{x\to 0}\left(\frac{\sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)}{\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)}\right) \cdot \frac{\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)}{\frac{\pi}{2}\sin x}$

$= \lim_{x\to 0} \frac{\pi}{6} \frac{\sin\left(\frac{\pi}{2}\sin x\right)}{\frac{\pi}{2}\sin x} = \frac{\pi}{6}$

Again, $gof(x) = 1$

$\Rightarrow \quad \frac{\pi}{2}\sin\left(\sin\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right) = 1$

$\Rightarrow \quad \sin\left(\sin\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right) = \frac{2}{\pi} > \frac{1}{2}$

So no x is possible.

24. (d): Put $x = 3\sin^2\theta - \cos^2\theta \Rightarrow dx = 8\sin\theta\cos\theta\, d\theta$

(P) $I = \int_0^{\frac{\pi}{2}} \frac{8\sin\theta\cos\theta d\theta}{4\sin\theta\cos\theta} = 2\int_0^{\frac{\pi}{2}} d\theta = \pi$

(Q) $I = \frac{1}{8}\int_0^{\frac{\pi}{2}} \frac{\sin\theta}{\cos\theta} \cdot 8\sin\theta\cos\theta\, d\theta = \int_0^{\frac{\pi}{2}} \sin^2\theta\, d\theta$

$= \frac{1}{2} \cdot \frac{\pi}{2} = \frac{\pi}{4}$

(R) $I = \frac{1}{4}\int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sin\theta} \cdot 8\sin\theta\cos\theta\, d\theta = 2 \cdot \frac{1}{2} \cdot \frac{\pi}{2} = \frac{\pi}{2}$

(S) $\int_0^{\frac{\pi}{2}} 4\sin\theta\cos\theta \cdot 8\sin\theta\cos\theta\ d\theta$

$= 32\int_0^{\frac{\pi}{2}} \sin^2\theta\cos^2\theta\, d\theta = 32 \cdot \frac{1}{4 \cdot 2} \cdot \frac{\pi}{2} = 2\pi.$

25. (b)

26. (0): $a_k = 2a_{k-1} - a_{k-2} \Rightarrow a_{k-2} + a_k = 2a_{k-1}$

$\therefore \quad a_1, a_2, \ldots\ldots\ldots, a_{11}$ form an A.P.

$a_r = a_1 + (r - 1)d = 15 + (r - 1)d$

$\therefore \quad \frac{1}{11}\sum_{r=1}^{11} a_r^2 = \frac{1}{11}\sum_{r=1}^{11} (225 + (r-1)^2 d^2 + 30(r-1)d)$

$= 225 + \frac{10 \cdot 21}{6} d^2 + \frac{30 \cdot 10d}{2} = 90$

$\Rightarrow \quad 35d^2 + 150d + 135 = 0 \Rightarrow 7d^2 + 30d + 27 = 0$

Solving, we get, $d = -3, \frac{-9}{7}$

and $a_2 = 15 + d = 15 - \frac{9}{7} = \frac{96}{7} \Rightarrow \quad 2a_2 = \frac{192}{7} \not< 27$

$\therefore \quad d = -3 \Rightarrow \frac{1}{11}(a_1 + a_2 + ... + a_{11})$

$= \frac{1}{2}(2a_1 + 10d) = \frac{1}{2}(30 - 30) = 0$

27. (1): The equation of the family of curves passing through the point of intersection of $x^2 - y^2 = a^2$ and $y = x^2$ is $x^2 - y^2 - a^2 + \lambda(x^2 - y) = 0$

$\Rightarrow \quad x^2(1 + \lambda) - y^2 - a^2 - \lambda y = 0$

It will be a circle if $1 + \lambda = -1$ *i.e.*, $\lambda = -2$.

Therefore, $-x^2 - y^2 - a^2 + 2y = 0$

$\Rightarrow \quad x^2 + y^2 - 2y = -a^2 \Rightarrow x^2 + (y - 1)^2 = 1 - a^2$

$\Rightarrow \quad 1 - a^2 > 0 \Rightarrow a^2 < 1$ ...(i)

Also both curves intersect at real points if $y^2 - y + a^2 = 0$ for some real y.

So if $-\frac{1}{2} \le a \le \frac{1}{2}$ ...(ii)

(i) and (ii) gives $a \in \left(-\frac{1}{2}, \frac{1}{2}\right)$

28. (8): We have, $(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$ ...(i)

Replacing x by $\frac{2}{x}$, we get

$$\left(\frac{8}{x^2} + \frac{6}{x} + 4\right)^{10} = \sum_{r=0}^{20} a_r \left(\frac{2}{x}\right)^r$$

$$\Rightarrow \quad \frac{2^{10}(2x^2 + 3x + 4)^{10}}{x^{20}} = \sum_{r=0}^{20} a_r 2^r x^{-r}$$

$$\Rightarrow \quad \sum_{r=0}^{20} a_r x^r = \sum_{r=0}^{20} a_r 2^{r-10} x^{20-r} \quad \text{[Using (i)]}$$

On comparing the coefficient of x^7 from both sides, we get, $a_7 = a_{13} 2^3 \Rightarrow \frac{a_7}{a_{13}} = 8$.

29. (1): Given, $\frac{\sqrt{2}\sin\alpha}{\sqrt{1 + \cos 2\alpha}} = \frac{1}{7}$

and $\sqrt{\frac{1 - \cos 2\beta}{2}} = \frac{1}{\sqrt{10}}$

$\Rightarrow \quad \frac{\sqrt{2}\sin\alpha}{\sqrt{2}\cos\alpha} = \frac{1}{7}$ and $\frac{\sqrt{2}\sin\beta}{\sqrt{2}} = \frac{1}{\sqrt{10}}$

$\Rightarrow \quad \tan\alpha = \frac{1}{7}$ and $\sin\beta = \frac{1}{\sqrt{10}}$

Now, $\tan\beta = \frac{1}{3}$ and $\tan 2\beta = \frac{2 \cdot \frac{1}{3}}{1 - \frac{1}{9}} = \frac{\frac{2}{3}}{\frac{8}{9}} = \frac{3}{4}$

$\therefore \quad \tan(\alpha + 2\beta) = \frac{\tan\alpha + \tan 2\beta}{1 - \tan\alpha\tan 2\beta}$

$= \frac{\frac{1}{7} + \frac{3}{4}}{1 - \frac{1}{7} \cdot \frac{3}{4}} = \frac{\frac{4 + 21}{28}}{\frac{25}{28}} = 1$

30. (2): Let $g(x) = \int_x^{\pi/2} \left[f'(t)\operatorname{cosec} t - \cot t \operatorname{cosec} t\, f(t) \right] dt$

$= \int_x^{\pi/2} \frac{d}{dt}(f(t)\operatorname{cosec} t)$

So, $g(x) = f(\pi/2)\operatorname{cosec}\frac{\pi}{2} - f(x)\operatorname{cosec} x$

$= 3 - f(x)\operatorname{cosec} x$

$\therefore \quad g(x) = 3 - \frac{f(x)}{\sin x}$

$\therefore \quad \lim_{x \to 0} g(x) = 3 - \lim_{x \to 0} \frac{f'(x)}{\cos x} = 3 - f'(0) = 3 - 1 = 2$

Brain Teaser

LOGICAL REASONING

for Various Competitive Exams

Direction Q.(1 and 2) : Find the wrong term in the given series:

1. 0, 2, 3, 9, 8, 30
(a) 9 (b) 8 (c) 30 (d) 2

2. AZ25, DW19, GT10, JQ7, MN1
(a) AZ25 (b) DW19 (c) GT10 (d) JQ7

Direction Q.(3 and 4) : The two words or numbers on the right of (: :) are related in the some way as the words on the left of (: :). Identify what will come in place of blank?

3. BDGK : YWTP :: ACFJ :...............
(a) WURN (b) ZXUQ (c) WTPK (d) ZWSN

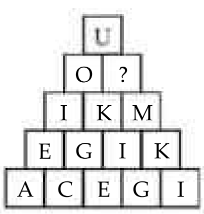
4. 439 : 39 :: 298 : ....
(a) 73 (b) 25 (c) 68 (d) 29

Direction Q.(5 and 6) : Three of the following four are alike in a certain way and so form a group. Which is the one that does not belong to that group?

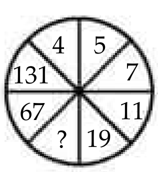
5.
(a) Firm (b) Grave
(c) Agreeable (d) Fastidious

6.
(a) Grovel (b) Squirm
(c) Writhe (d) Tarry

Direction Q.(7 and 8): In each of the following figures, find the number/letter which replaces the sign of '?'.

7.


(a) P (b) S (c) R (d) Q

8.


(a) 36 (b) 32 (c) 34 (d) 35

Directions Q.(9 and 10): Study the following information to answer the given questions.

9. @ + # = 5, # + % = 7, % + ^ = 9, ^ + @ = 7, then the value of $(@ + \# + \% + \wedge)^2$ is
(a) 144 (b) 196 (c) 256 (d) 361

10. If 5 = 7, 9 = 5, 13 = 3, 17 = 1 and 21 = –1, then 25 equals
(a) –5 (b) –3 (c) –7 (d) –9

Direction Q.(11 and 12): Read information carefully and answer the following questions:

If 'A + B' means 'A is the father of B'.
If 'A × B' means 'A is the sister of B'.
If 'A $ B' means 'A is the wife of B'.
If 'A % B' means 'A is the mother of B'.
If 'A ÷ B' means 'A is the son of B'.

11. What should come in place of the question mark (?) to establish that J is the brother of T in the expression?
J ÷ P % H ? T % L
(a) × (b) ÷
(c) $ (d) Either ÷ or ×

12. Which among the following expressions is true if Y is the grandson of X.
(a) W % L × T × Y ÷ X
(b) W + L × T × Y ÷ X
(c) X + L × T × Y ÷ W
(d) W $ X + L + Y + T

13. Four aeroplanes of Airforce viz., A, B, C and D started for a demonstration flight towards East. After flying 50 km, planes A and D flew towards right, planes B and C flew towards left. After 50 km, planes B and C flew towards their left, planes A and D towards their left. In which directions the aeroplanes A, B, D, C respectively fly now?
(a) North, South, East, West
(b) South, North, West, East
(c) East, West, East, West
(d) West, East, West, East

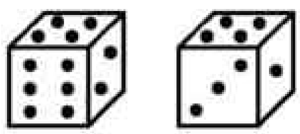
Direction Q.(14 and 15): In these questions, relationship between different elements is shown in the statements. These statements are followed by two conclusions.
Mark answer

(a) if only conclusion (I) is true
(b) if only conclusion (II) is true
(c) if either conclusion (I) or (II) is true
(d) if neither conclusion (I) nor (II) is true

14. **Statement :** $U > A = I \leq O < E$
Conclusions : I. $I \leq E$ II. $O > U$

15. **Statements :** $L = M \geq N$; $L > P$; $N > K$
Conclusions : I. $K > P$ II. $M > K$

16. From the given two positions of a single dice, find the number of dots at top face of the dice, if its bottom face has 6 dots.



I II

(a) 1 (b) 2 (c) 4 (d) 3

Direction Q.(17 to 20): Study the following information carefully and answer the questions given below:

A group of seven friends, A, B, C, D, E, F and G work as Economist, Agriculture Officer, IT Officer, Terminal Operator, Clerk, Forex Officer and Research Analyst, for Banks L, M, N, P, Q, R and S but not necessarily in the same order. C works for Bank N and is neither a Research Analyst nor a Clerk. E is an IT Officer and works for Bank R. A works as Forex Officer and does not work for Bank L or Q. The one who is an Agriculture Officer works for Bank M. The one who works for Bank L works as a Terminal Operator. F works for Bank Q. G works for Bank P as a Research Analyst. D is not an Agriculture Officer.

17. Who amongst the following works as an Agriculture Officer?
(a) C (b) B (c) F (d) D

18. What is the profession of C?
(a) Terminal operator (b) Agriculture Officer
(c) Economist (d) Research Analyst

19. For which Bank does B work?
(a) M (b) S
(c) L (d) Either M or S

20. What is the profession of the person who works for Bank S?
(a) Clerk (b) Agriculture Officer
(c) Terminal Operator (d) Forex Officer

SOLUTIONS

1. **(c) :** $1^2 - 1 = 0$, $1^3 + 1 = 2$, $2^2 - 1 = 3$, $2^3 + 1 = 9$
$3^2 - 1 = 8$, $3^3 + 1 = 28$.
Wrong term 30.

2. **(c) :** Place value of Z – Place value of A $\Rightarrow$ 26 – 1 = 25
Place value of W – Place value of D $\Rightarrow$ 23 – 4 = 19
Place value of T – Place value of G $\Rightarrow$ 20 – 7 = $\boxed{10}$
Place value of Q – Place value of J $\Rightarrow$ 17 – 10 = 7
Place value of N – Place value of M $\Rightarrow$ 14 – 13 = 1
Thus, wrong term is GT10.

3. **(b) :** Write A to Z and Z to A and replace each letter by the corresponding letter.

4. **(b) :** 439 = (4 × 9) + 3, the relation is (first digit × last digit) + middle digit.

5. **(b) :** As other options are synonyms of each other.

6. **(d) :** Tarry means to be covered in tar. All others are synonyms of each other.

7. **(d) :** The figure consist of letter with the left most letter of each row being a vowel. This gives us a hint that there is a logic which exists row wise checking each row.

5^{th} row $\Rightarrow A \xrightarrow{+2} C \xrightarrow{+2} E \xrightarrow{+2} G \xrightarrow{+2} I$

Hence, each letter is 2 places ahead of the previous letter, while moving from left to right. We first check the last row because it contains the maximum number of letters and hence is easier to find the logic.
Now, checking the same logic on 4^{th} and 3^{rd} rows.

4^{th} row $\Rightarrow E \xrightarrow{+2} G \xrightarrow{+2} I \xrightarrow{+2} K$

3^{rd} row $\Rightarrow I \xrightarrow{+2} K \xrightarrow{+2} M$

Hence, the logic is verified and now we apply the same logic to 2^{nd} row.

2^{nd} row $\Rightarrow O \xrightarrow{+2} Q$

So, the value of ? is Q.

8. **(d) :** $4 + 2^0 = 5$; $5 + 2^1 = 7$; $7 + 2^2 = 11$
$11 + 2^3 = 19$; $19 + 2^4 = 35$.

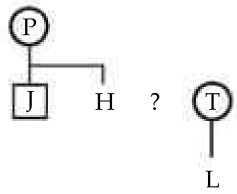
9. **(b) :** Adding all we get, 2(@ + # + % + ^) = 28
@ + # + % + ^ = 14
Hence, $(@ + \# + \% + \hat{})^2 = 196$.

10. **(b) :**

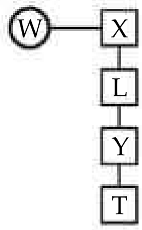
	5	=	7	
+4				–2
	9	=	5	
+4				–2
	13	=	3	
+4				–2
	17	=	1	
+4				–2
	21	=	–1	

Hence, 25 = – 3.

11. (a) : Drawing family tree from given information:
'×' is used to establish that J is brother of T in the expression.

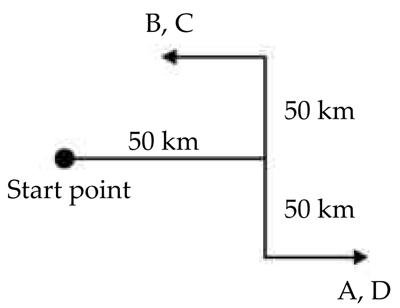


12. (d) : W \$ X + L + Y + T can be drawn as



Thus, X is grandfather of Y.

13. (c) :



14. (d) : Statement : $U > A = I \leq O < E$
Conclusions: I. $I \leq E$ II. $O > U$

15. (b) : Statements : $L = M \geq N$; $L > P$; $N > K$
On combining all the statements, we get,
$P < L = M \geq N > K$
Conclusions: I. $K > P$ II. $M > K$
Hence, only conclusion (II) is definitely true.

16. (d) : In both the positions, 4 dots are common at the same (top) face. So, face having 6 dots will be opposite to the face having 3 dots.

(17 - 20) :

Person	Profession	Banks
A	Forex Officer	S
B	Agriculture Officer	M
C	Economist	N
D	Terminal Officer	L
E	IT Officer	R
F	Clerk	Q
G	Research Analyst	P

17. (b) **18.** (c) **19.** (a) **20.** (d)

◈◈

THE ABEL PRIZE 2023

The most coveted prize in mathematics, the so-called Mathematics Nobel 2023 has been announced

ARGENTINIAN-BORN mathematician Luis Caffarelli, 74, has won the 2023 Abel Prize for his work on partial differential equations which, by relating one or more unknown functions and their derivatives, allow scientists to predict the behaviour of the physical world.

The Prize

First awarded in 2003, the Abel Prize "recognises pioneering scientific achievements in mathematics". It is named after Norwegian mathematician Niels Henrik Abel (1802-29) and is often considered to be an equivalent of the Nobel Prize, which does not have a category for mathematics.

The Abel Prize was proposed in 1899 by the Norwegian mathematician Sophus Lie when he learned that Alfred Nobel's plans for annual prizes did not include a prize in mathematics. The Norwegian Parliament finally established the prize in 2002 to mark Abel's 200th birth anniversary.

The recipients are chosen by the Abel Committee, which comprises mathematicians appointed by the Norwegian Academy of Science and Letters (which awards and administers the Abel Prize on behalf of the Norwegian government) on the advice of the International Mathematical Union (IMU) and the European Mathematical Society (EMS).

The prize includes an award of 7.5 million kroner (roughly \$720,000) and a glass plaque designed by the Norwegian artist Henrik Haugan. Till date, 26 mathematicians have been awarded the Abel Prize. Notably, while it is modelled after the Nobels, the Abel Prize is often seen as recognition of a lifetime achievement in mathematics.

The Winner

Caffarelli was born and raised in Buenos Aires, Argentina, and is currently professor at the University of Texas, Austin, He has been one of the leading figures in the study of partial differential equations for over five decades.

Luis A. Caffarelli

Partial differential equations arise naturally as laws of nature, to describe phenomena as different as the flow of water or the growth of populations. They have been studied since the time of Isaac Newton and Gottfried Leibniz; however, several key problems continue to elude solutions.

Caffarelli has been honoured ''for his seminal contributions to regularity theory for nonlinear partial differential equations including free-boundary problems and the Monge-Ampère equation", the Academy of Science and Letters said in its citation.

Caffarelli's "groundbreaking contributions" have "radically changed our understanding of classes of nonlinear partial differential equations with wide applications", says the citation. "The tools the Caffarelli has come up with have been applied to many different problems, from equations describing nature to financial mathematics", Helge Holden, chair of the Abel committee, said.

QUANTITATIVE APTITUDE

For Various Competitive Exams

1. If $\frac{37}{13}=2+\cfrac{1}{x+\cfrac{1}{y+\cfrac{1}{z}}}$, where $x, y, z \in N$, then value of $x^2+y^2+z^2$ equals

(a) 1 (b) 4 (c) 25 (d) 30

2. A person invested partly ₹ 45000 at the rate 4% and 6% respectively. If his annual income from both investments are equal, then the average rate of interest is

(a) 5.2% (b) 5.0% (c) 4.7% (d) 4.8%

3. My brother 4 years elder to me. My father was 30 years of age when my sister was born while my mother was 27 years of age when I was born. If my sister was 5 years of age when my brother was born, then what was the age of my father and mother respectively, when my brother was born?

(a) 35, 31 (b) 32, 29 (c) 35, 23 (d) 29, 23

4. If $\sqrt[n]{\frac{9^{\left(n+\frac{1}{4}\right)}\times\sqrt{3\cdot 3^{-n}}}{3\cdot\sqrt{3^{-n}}}}=x$ then value of x equals

(a) 3^3 (b) 3^2 (c) $\sqrt[n]{3}$ (d) 3

5. A screwdriver and a hammer cost the same. If the price of screwdriver is increased by 5% and that of hammer by 3%, how much more will it to buy 3 screwdrivers and 3 hammers?

(a) 24% (b) 4% (c) 12% (d) 8%

6. A trader mark 10% higher the C.P. He gives a discount of 10% on M.P. In this kind of sales, how much % does the trader gain/loss?

(a) 5% gain (b) 5% loss
(c) 1% gain (d) 1% loss

7. A cat takes 7 leaps for every 4 leaps of a dog, but 4 leaps of dog is equal to 5 leaps of cat. What is the ratio of speeds of cat to that of dog?

(a) 7 : 4 (b) 5 : 4 (c) 7 : 5 (d) 6 : 5

8. *A* and *B* started a business with ₹ 20000 and ₹ 35000 respectively. They agreed to share the profit in the ratio of their capital. *C* joins the partnership with the condition that *A*, *B* and *C* will share profit equally and pays ₹ 220000 as premium for this, to be shared between *A* and *B*. This is to be divided between *A* and *B* in the ratio of

(a) 10 : 9 (b) 1 : 10 (c) 10 : 1 (d) 9 : 10

9. If the cost of printing a book of 320 leaves with 21 lines on each page and on an average 11 words in each line is ₹ 19, then the cost of printing a book with 297 leaves, 28 lines on each page & 10 words in each line is

(a) ₹ $21\frac{3}{8}$ (b) ₹ $20\frac{3}{8}$ (c) ₹ $21\frac{1}{8}$ (d) ₹ $18\frac{2}{3}$

10. The workdone by a girl in 8 hours is equal to the work done by a boy in 6 hours and by a man in 4 hours. If working 6 hours per day 10 men can complete a work in 15 days, then in how many days 15 boys, 12 girls and 4 men together finish the same work working 5 hours per day?

(a) 12 (b) 13 (c) 10 (d) 9

11. If *A*, *B* and *C* contract a work for ₹ 510. If work of *A* and *C* together supposed to do 12/17 of the whole work. What is the amount goes to *B*?

(a) ₹ 370 (b) ₹ 360 (c) ₹ 150 (d) ₹ 140

12. A tank has a leak which would empty it in 12 hours. A tap is turned on which admits 5 litres/minute into the tank and it is now emptied in 16 hours. What is the capacity of the tank?

(a) 7200 litres (b) 14400 litres
(c) 2880 litres (d) 1440 litres

13. A 320 metre long train crosses a pole in 16 seconds. It stops five times of duration 18 minutes each. What time will it take in covering a distance of 576 km?

(a) 9 hours (b) $9\frac{1}{4}$ hours
(c) $9\frac{1}{2}$ hours (d) $8\frac{1}{2}$ hours

14. Several litres of acid were drawn off from a 48 litre vessel full of acid and equal amount of water was added. Again the same volume of the mixture was drawn off and replaced by water. As a result now the vessel contains 27 litres of pure acid. How much of the acid was drawn off initially?

(a) 18 litres (b) 12 litres

(c) 9 litres (d) 6 litres

15. A sum of money ₹ 1550 was lent partly at 5% and partly at 8% simple interest. The total interest received in 3 years was ₹ 300. What is the ratio of money lent at 5%, 8% respectively?

(a) 11 : 12 (b) 5 : 8 (c) 16 : 15 (d) 8 : 5

16. An amount is invested in a bank at compound rate of interest. The total amount including interest after first year and third year is ₹ 1200 and ₹ 1587 respectively. What is the rate of interest?

(a) 10% (b) 12% (c) 15% (d) 20%

17. If $\log_{15}\log_{21}(\sqrt{x+21}+\sqrt{x})=0$, then $x=$

(a) 21 (b) 15

(c) 91 (d) None of these

18. A square park having each side 75 m. At each corner of the park, there is a flower bed in the form of a quadrant of radius 9 m and in the middle of square there is circular bed whose radius is 4 metre, the remaining area to be grassed at the rate of ₹ 1.40 per square metre. The cost of grassing is

(a) ₹ 3724.10 (b) ₹ 7448.20

(c) ₹ 2369.88 (d) None of these

19. A cube has edge 3 cm and cuboid is 2 cm long, 3 cm wide and 3 cm high, the paint in a certain container is sufficient to paint an area equal to 120 cm^2 then which one of the following is correct?

(a) only cuboid can be painted

(b) neither cube nor cuboid can be painted

(c) only cube can be painted

(d) both cube and cuboid can be painted

20. *A* can run 1 km in 4 min 40 secs while *B* takes 5 mins to complete the race. How many metre's start can *A* give *B* in 1 km race so that the race may end in a dead heat?

(a) $33\frac{1}{3}$ m (b) $33\frac{2}{3}$ m

(c) $66\frac{2}{3}$ m (d) None of these

SOLUTIONS

1. (d) : From given

$$\frac{37}{13}=2\frac{11}{13}=2+\frac{11}{13}=2+\cfrac{1}{x+\cfrac{1}{y+\cfrac{1}{z}}}$$

$$\Rightarrow \frac{11}{13}=\cfrac{1}{x+\cfrac{1}{y+\cfrac{1}{z}}} \quad \Rightarrow x+\cfrac{1}{y+\cfrac{1}{z}}=\frac{13}{11}$$

$$\Rightarrow x+\cfrac{1}{y+\cfrac{1}{z}}=1+\frac{2}{11} \quad \Rightarrow x=1$$

Now, $\cfrac{1}{y+\cfrac{1}{z}}=\frac{2}{11} \Rightarrow y+\frac{1}{z}=\frac{11}{2}=5+\frac{1}{2}$

$\Rightarrow y = 5$ and $z = 2$

∴ x, y, z are respectively 1, 5, 2

∴ $x^2 + y^2 + z^2 = 1^2 + 5^2 + 2^2 = 30.$

2. (d) : Let investment at the rate 4% be ₹ x.

∴ The investment at the rate 6% = ₹ $(45000 - x)$

As annual income from both investment are equal

∴ $\frac{x\times 4}{100}=\left(\frac{45000-x}{100}\right)\times 6 \Rightarrow 10x=270000$

$\Rightarrow x =$ ₹ 27000 and other part be ₹ 18000

∴ Income from ₹ 27000 = 270 × 4 = ₹ 1080

and income from ₹ 18000 = 180 × 6 = ₹ 1080

∴ Total income = 1080 × 2 = ₹ 2160

$$\text{Average rate of interest}=\frac{\text{Total interest}}{\text{Total capital}}\times 100$$

$$=\frac{216}{45}=4.8\%$$

3. (c) : Clearly, my brother was born 4 years ago I was born and 5 years after my sister was born.

∴ Father's age when brother was born

= (30 + 5) years = 35 years

Mother's age when brother was born

= (27 – 4) years = 23 years

4. (b) : $\sqrt[n]{\frac{9^{\left(n+\frac{1}{4}\right)}\times\sqrt{3\times 3^{-n}}}{3\times\sqrt{3^{-n}}}}=x$

$$\Rightarrow \sqrt[n]{\frac{9^n\times 9^{\frac{1}{4}}\times\sqrt{3}\times\sqrt{3^{-n}}}{3\times\sqrt{3^{-n}}}}=x \Rightarrow \sqrt[n]{\frac{9^n\times\sqrt{3}\times\sqrt{3}}{3}}=x$$

$\Rightarrow (9^n)^{1/n} = x \Rightarrow 9 = x$ or $x = 3^2$

5. **(b)**

6. **(d) :** Let the cost price be ₹ x.

$\therefore$ M.P. (marked price is 10% above the C.P.)

$$= ₹\frac{x \times 110}{100} = ₹\frac{11x}{10}$$

S.P. (10% discount on M.P.)

$$= ₹\frac{11x}{10} \times \frac{90}{100} = ₹\frac{99x}{100} < ₹\, x$$

Here C.P. > S.P., which shows the loss in the transaction.

$$\therefore \quad \text{Loss} = \text{C.P.} - \text{S.P.} = ₹\frac{x}{100}$$

$$\therefore \quad \text{Loss}\ \% = \frac{\frac{x}{100}}{x} \times 100 = 1\%$$

7. **(c) :** As 5 leaps of cat = 4 leaps of dog

$\therefore$ 1 leap of cat = 4/5 leap of dog

Now, for every 7 leaps of cat dog take 4 leaps

$\therefore$ 7 × cat's leap : 4 × dog's leap

$$= 7 \times \frac{4}{5}\text{ dog's leap} : 4 \times \text{dog's leap} = 7 : 5$$

8. **(c) :** Ratio of total capital of A and B

= 20000 × 12 : 35000 × 12

= 240000 : 420000

Now, C gives ₹ 220000 to A and B so that they can make their capital equal

$\therefore$ A's capital : B's capital = 240000 : 420000

+ 200000 : 20000

= 440000 : 440000

$\therefore$ Required ratio of divided amount

= 200000 : 20000 = 10 : 1

9. **(a) :** Let the cost of printing the book be ₹ x.

Less leaves, less cost (Direct proportion)

More lines, more cost (Direct proportion)

Less words, less cost (Direct proportion)

$$\left.\begin{matrix}\text{Leaves} & 320 : 297\\ \text{Lines} & 21 : 28\\ \text{Words} & 11 : 10\end{matrix}\right\} :: 19 : x$$

$\Rightarrow$ 320 × 21 × 11 × x = 297 × 28 × 10 × 19

$$\Rightarrow x = \frac{297 \times 28 \times 10 \times 19}{320 \times 21 \times 11} = \frac{27 \times 7 \times 10 \times 19}{80 \times 21}$$

$$= \frac{171}{8} = ₹\, 21\frac{3}{8}$$

10. **(d) :** Given, 4 working hours of man

= 6 working hours of a boy

= 8 working hours of a girl

$\Rightarrow$ $4M = 6B = 8G$ ....(i)

$\therefore$ $4M + 15B + 12G$

$= 4M + 10M + 6M = 20M$ (using (i))

Let D_2 be the required number of days.

Now using, $\frac{M_1 D_1 T_1}{w_1} = \frac{M_2 D_2 T_2}{w_2}$

where $w_1 = w_2 = 1$ work

$10M \times 15 \times 6 = 20M \times D_2 \times 5$

($\because$ $M_1 = 10M$, $D_1 = 15$ days, $T_1 = 6$ hours)

$M_2 = 4M + 15B + 12G = 20M$, $T_2 = 5$ hours

$\Rightarrow$ $D_2 = 9$ days

11. **(c) :** As A and C together do the work = 12/17

$$\therefore \quad \text{Work done by } B = 1 - \frac{12}{17} = \frac{5}{17}$$

$$\therefore \quad (A + C)\text{'s share} : B\text{'s share} = \frac{12}{17} : \frac{5}{17} = 12 : 5$$

Total contract rate = ₹ 510

$$\therefore \quad B\text{'s share} = ₹\frac{5}{17} \times 510 = ₹\, 150$$

12. **(b) :** Inlet and leak emptied the tank in 16 hours

Leak emptied the tank in 12 hours.

MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics. In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is combined using a specified operation on its numbers. For example, if that value is 3 for a two-box cluster and operation is multiply, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.

9+			1–	3–	2–
6+		8+			
3–	7+			3–	
		12+		4–	
4–			1–		8+
14+			5+		

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

∴ Work done by inlet in 1 hour

$$= \frac{1}{12} - \frac{1}{16} = \frac{4-3}{48} = \frac{1}{48}$$

∴ Work done by inlet in 1 min $= \frac{1}{48 \times 60}$

Now volume of $\frac{1}{48 \times 60}$ part = 5 litres

∴ Volume of full part = 5 × 48 × 60 litres
= 14400 litres

13. (c) : Speed of train $= \frac{320}{16} = 20 \text{ m/s}$

or $20 \times \frac{18}{5} = 72 \text{ km/hr}$

Time taken to cover 576 km $= \frac{576}{72} = 8$ hours

But the train stops for 5 times, 18 minutes each *i.e.*

18 × 5 = 90 minutes = $1\frac{1}{2}$ hours

Total time taken = 8 + $1\frac{1}{2}$ hours = $9\frac{1}{2}$ hours

14. (b) : Let a container have x units of acid initially and y units of acid is drawn from it. If operation be repeated 'n' times; then the quantity of acid in the container is $x\left(1-\frac{y}{x}\right)^n$.

In this question, process is repeated twice

∴ n = 2 and quantity of pure acid is 27 litres, x = 48 litres, y = ?

$$\therefore\ 48\left(1-\frac{y}{48}\right)^2 = 27 \Rightarrow \left(1-\frac{y}{48}\right)^2 = \frac{27}{48} = \frac{9}{16} = \left(\frac{3}{4}\right)^2$$

$$\Rightarrow 1-\frac{y}{48} = \frac{3}{4} \Rightarrow \frac{y}{48} = \frac{1}{4} \Rightarrow y = 12 \text{ litres}$$

∴ Acid drawn off = 12 litres

15. (c) : Let the sum lent at 8% be ₹ x.

∴ Sum lent at 5% = ₹(1550 – x)

Now according to question, we have

$$\frac{8 \times x \times 3}{100} + \frac{5 \times (1550 - x) \times 3}{100} = 300$$

⇒ $24x - 15x + 15 \times 1550 = 300 \times 100$

⇒ $9x = 30000 - 23250 = 6750 \Rightarrow x = \frac{6750}{9} = 750$

∴ Required ratio (5% to 8%) = 800 : 750 *i.e.* 16 : 15

16. (c) : Let the amount be ₹ x.

and rate of interest = r% p.a.

According to problem, we have

Amount after one year = ₹ 1200

$$\therefore\ x\left(1+\frac{r}{100}\right) = 1200 \quad \text{...(i)}$$

Again, amount after 3rd year = ₹ 1587

$$\therefore\ x\left(1+\frac{r}{100}\right)^3 = 1587 \quad \text{...(ii)}$$

On dividing (ii) by (i), we have

$$\left(1+\frac{r}{100}\right)^2 = \frac{1587}{1200} = \frac{529}{400} = \left(\frac{23}{20}\right)^2$$

$$\Rightarrow \left(1+\frac{r}{100}\right) = \frac{23}{20} \Rightarrow \frac{r}{100} = \frac{3}{20} \Rightarrow r = 15\%$$

17. (d) : $\log_{15}\log_{21}(\sqrt{x+21}+\sqrt{x}) = 0$

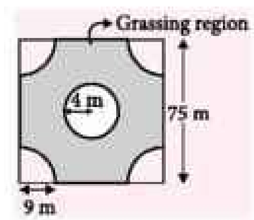
$\Rightarrow \log_{21}(\sqrt{x+21}+\sqrt{x}) = 15^0 = 1$

$\Rightarrow \sqrt{x+21}+\sqrt{x} = 21^1 \Rightarrow \sqrt{x+21} = 21 - \sqrt{x}$

Squaring both sides, we get

$441 - 21 = 42\sqrt{x} \Rightarrow \sqrt{x} = 10 \Rightarrow x = 100$

18. (b) : Area of square park
= 75 × 75 = 5625 m^2



Area covered by flower bed region
= Area of 4 quadrants + Area of circle

$$= 4 \times \frac{\pi}{4}(9)^2 + \pi(4)^2 = \pi\ (81 + 16) \text{ sq. m}$$

$$= \frac{97 \times 22}{7} \text{ sq. m} = \frac{2134}{7} \text{ sq. m}$$

∴ Area for grassing (shaded area)

$$= \left(5625 - \frac{2134}{7}\right) \text{sq.m} = \frac{37241}{7} \text{sq.m}$$

$$\therefore \text{ Cost of grassing} = \frac{37241}{7} \times 1.40 = \frac{37241}{5} = ₹\ 7448.20$$

19. (d) : Area of cube of = $6(3)^2$ = 54 sq. cm

Area of cuboid of dimension 2 cm × 3 cm × 3 cm
= 2(2 × 3 + 3 × 3 + 2 × 3)sq. cm
= 2(6 + 9 + 6) sq. cm = 42 sq. cm

∴ Total area of cube and cuboid collectively
= 54 + 42 = 96 sq. cm < 120 sq. cm

∴ Both cube and cuboid can be painted.

20. (c) : A beats B by (300 – 280) = 20 secs

∴ Distance covered by B in 5 mins *i.e.*, 300 secs = 1000 m.

∴ Distance covered by B in 20 secs

$$= \frac{1000}{300} \times 20 = \frac{200}{3} = 66\frac{2}{3} \text{ m}$$

So, A can give B a start of $66\frac{2}{3}$ m in 1 km race so that the race end in a dead heat.

YOU ASK WE ANSWER

Do you have a question that you just can't get answered?
Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. $\bar{b}z + b\bar{z} = c$ is the equation of a straight line. If z_1 and z_2 be mirror image of each other in this line, then prove that $\bar{b}z_2 + b\bar{z}_1 = c$.

(Jyoti, Haridwar)

Ans. If z_1 and z_2 are mirror image of each other in the line

$\bar{b}z + b\bar{z} = c$ ...(i)

then the mid-point of z_1, z_2 must lie on (i) ...(ii)

and the line joining z_1, z_2 must be perpendicular to (i) ...(iii)

Satisfying condition (ii), we have

$$\bar{b}\left(\frac{z_1+z_2}{2}\right) + b\left(\overline{\frac{z_1+z_2}{2}}\right) = c$$

i.e., $(\bar{b}z_1 + b\bar{z}_2) + (\bar{b}z_2 + b\bar{z}_1) = 2c$ ...(iv)

Putting $z = x + iy$ in equation (i), it reduces to

$(\bar{b} + b)x + i(\bar{b} - b)y - c = 0$,

whose slope $= \dfrac{(b+\bar{b})}{i(b-\bar{b})}$

Equation of the line joining z_1, z_2 is

$$\begin{vmatrix} z & \bar{z} & 1 \\ z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \end{vmatrix} = 0$$

i.e., $z\bar{a} - \bar{z}a + d = 0$

where $a = z_1 - z_2$ and $d = z_1\bar{z}_2 - \bar{z}_1 z_2$

Putting $z = x + iy$, the above equation reduces to

$(\bar{a} - a)x + i(\bar{a} + a)y + d = 0$,

whose slope $= \dfrac{(a-\bar{a})}{i(a+\bar{a})}$

Satisfying equation (iii), we have

$$\frac{(b+\bar{b})}{i(b-\bar{b})} \times \frac{(a-\bar{a})}{i(a+\bar{a})} = -1$$

$\Rightarrow \quad (b+\bar{b})(a-\bar{a}) = (b-\bar{b})(a+\bar{a}) \Rightarrow \bar{b}a = b\bar{a}$

$\Rightarrow \quad \bar{b}z_1 + b\bar{z}_2 = \bar{b}z_2 + b\bar{z}_1$ ...(v)

From equations (iv) and (v), we have

$\bar{b}z_2 + b\bar{z}_1 = c$,

which is the desired result.

2. Show that the integral part of $(5\sqrt{5} + 11)^{67}$ is even.

(Khushi, U.P.)

Ans. Let $(5\sqrt{5} + 11)^{67} = I + f$ where I and f are the integral and the fractional parts of $(5\sqrt{5} - 11)^{67}$ respectively.

Let $(5\sqrt{5} - 11)^{67} = g$ where g is a fraction. Since $0 < 5\sqrt{5} - 11 < 1$, therefore $0 < (5\sqrt{5} - 11)^{67} < 1$ for every positive integer n.

We have, $I + f - g = (5\sqrt{5} + 11)^{67} - (5\sqrt{5} - 11)^{67}$

$= 2[C_1(5\sqrt{5})^{66} \cdot 11 + C_3(5\sqrt{5})^{64} \cdot 11^3 + \ldots\ldots\ldots + C_{67} \cdot 11^{67}]$

[where $C_r = {}^{67}C_r$]

i.e., $I + f - g = 2k, k \in Z^+$ ...(i)

i.e., $f - g = 2k - I$ = an integer ...(ii)

Since $0 < f, g < 1$, therefore we have

$-1 < f - g < 1$ ...(iii)

Thus, using results (ii) and (iii), we have

$f - g = 0$

[since the only integral value in $(-1, 1)$ is 0] ...(iv)

Putting result (iv) in equation (i) we have

$I = 2k - (f - g) = 2k$ = an even integer.

3. What is the diameter of a circle inscribed in a regular polygon of 12 sides, each of length 1 cm ?

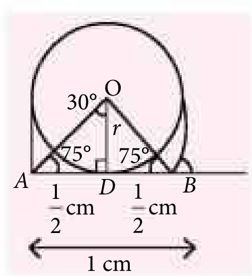
(Sunil, Delhi)

Ans. Exterior angle of polygon $= \dfrac{360°}{n} = \dfrac{360°}{12} = 30°$

So, Interior angle of polygon = 180° – 30° = 150°

OA bisects the interior angle, So,

$\angle OAD = 75°$



In $\Delta\, OAD$, $\tan 75° = \dfrac{OD}{AD}$

$\Rightarrow \tan(30° + 45°) = \dfrac{r}{1/2}$

$$\Rightarrow \frac{\tan 30° + \tan 45°}{1 - \tan 30° \tan 45°} = 2r$$

$$\Rightarrow \frac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}}} = 2r \quad \Rightarrow \frac{\sqrt{3}+1}{\sqrt{3}-1} = 2r \Rightarrow \frac{4+2\sqrt{3}}{2} = 2r$$

$\Rightarrow \quad d = 2 + \sqrt{3}$

CBSE warm-up!

CLASS-XI

Chapterwise practice questions for CBSE Exams as per the latest pattern and reduced syllabus by CBSE for the academic session 2023-24.

Series-1 Sets

Time Allowed : 3 hours
Maximum Marks : 80

General Instructions

(a) This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
(b) Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
(c) Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
(d) Section C has 6 Short Answer (SA)-type questions of 3 marks each.
(e) Section D has 4 Long Answer (LA)-type questions of 5 marks each.
(f) Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION - A (MULTIPLE CHOICE QUESTIONS)

Each question carries 1 mark.

1. If $A = \{x : x = 4|n| + 1, -5 \le n \le -2, n \in Z\}$, then number of subsets of A is
(a) 8 (b) 15 (c) 4 (d) 16

2. Which of the following sets are equal?
$A = \{x : x \in N, x < 3\}$; $B = \{1, 2\}$; $C = \{3, 1\}$;
$D = \{x : x \in N, x \text{ is odd}, x < 5\}$; $E = \{2, 1\}$; $F = \{1, 1, 3\}$
(a) $A = C = D$ (b) $A = B = E$
(c) $C = D = E$ (d) Both (b) and (c)

3. Which of the following is a finite set?
(a) Set of all points in a plane
(b) Set of all lines in a plane
(c) $\{x : x \in R \text{ and } 0 < x < 1\}$
(d) Set of all persons on the earth

4. Write all subsets of set $A = \{\phi, 1\}$.
(a) $\{\phi\}, \{1\}, \{\phi, 1\}$ (b) $\{\phi, \{\phi\}\}, \{1\}, \{\phi, 1\}$
(c) $\{\phi\}, \phi, \{1\}, \{\phi, 1\}$ (d) None of these

5. Let $A = \{x \in N : x \text{ is a multiple of } 3\}$ and $B = \{x \in N : x \text{ is a multiple of } 6\}$, then $A - B$ is equal to
(a) $\{6, 12, 18, ...\}$ (b) $\{3, 6, 9, 12, ...\}$
(c) $\{3, 9, 15, 21, ...\}$ (d) None of these

6. If $A = \{2, 3, 4, 8, 10\}$, $B = \{3, 4, 5, 10, 12\}$ and $C = \{4, 5, 6, 12, 14\}$, then $(A \cup B) \cap (A \cup C)$ is equal to
(a) $\{2, 3, 4, 5, 10, 12\}$ (b) $\{2, 3, 4, 5, 8, 10, 12\}$
(c) $\{2, 3, 4, 10, 12\}$ (d) None of these

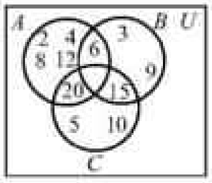
7. If $A = \{x : x = 3^n, n \le 6, n \in N\}$ and $B = \{x : x = 9^n, n \le 4, n \in N\}$, then $A \cap B$ is
(a) $\{3, 9, 27, 81, 243, 729, 6561\}$
(b) $\{9, 81, 729\}$
(c) $\{3, 27, 243\}$ (d) $\{3, 27, 243, 6561\}$

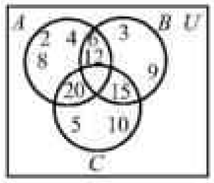
8. Let $A = \{(x, y) : y = e^{2x}\ \forall x \in R\}$ and $B = \{(x, y) : y = e^{-2x}\ \forall\ x \in R\}$, then $A \cap B$ is
(a) Not a set (b) Singleton set
(c) Empty set (d) None of these

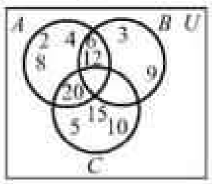
9. If A and B are two sets, then $(A \cup B')' \cap (A' \cup B)'$ is
(a) Null set (b) Universal set
(c) A' (d) B'

10. If A and B are non-empty sets, then $(A \cap B) \cup (A - B)$ is equal to
(a) B (b) A (c) A' (d) B'

11. Let A, B and C are subsets of universal set U. If $A = \{2, 4, 6, 8, 12, 20\}$, $B = \{3, 6, 9, 12, 15\}$, $C = \{5, 10, 15, 20\}$ and U is the set of all whole numbers. Then, the correct Venn diagram is

(a) 

(b) 

(c) 

(d) None of these

12. Which of the following is not an empty set?
(a) Set of natural numbers < 1
(b) Natural number between 3 and 4
(c) The set of integers between –2 and –3
(d) The set $A = \{x : x^2 = 2\ \forall\ x \in R\}$

13. Which one of the following have same sense w.r.t. open and closed intervals?
(a) $A = [2, 3]$ and $B = \{x : x \in R, 2 \le x \le 3\}$
(b) $A = [-3, 5]$ and $B = \{x : x \in R, -3 < x \le 5\}$
(c) $A = [-15, -2]$ and $B = \{x : x \in R, -15 < x < -2\}$
(d) $A = (-1, 2)$ and $B = \{x : x \in R, -1 \le x \le 2\}$

14. Which of the following is a universal set?
(a) Set of all points in xy-plane
(b) Set R of real numbers
(c) Set N of natural numbers
(d) All of these

15. The set builder form of given set $A = \{3, 6, 9, 12\}$ and $B = \{1, 4, 9, ..., 100\}$ is
(a) $A = \{x : x = 3n, n \in N$ and $1 \le n \le 5\}$, $B = \{x : x = n^2, n \in N$ and $1 \le n \le 10\}$
(b) $A = \{x : x = 3n, n \in N$ and $1 \le n \le 4\}$, $B = \{x : x = n^2, n \in N$ and $1 \le n \le 10\}$
(c) $A = \{x : x = 3n, n \in N$ and $1 \le n \le 4\}$, $B = \{x : x = n^2, n \in N$ and $1 < n < 10\}$
(d) None of these

16. Which of the following is a singleton set?
(a) $\{x : |x| < 1, x \in I\}$ (b) $\{x : |x| = 5, x \in I\}$
(c) $\{x : x^2 = 1, x \in I\}$ (d) $\{x : x^2 + x + 1 = 0, x \in R\}$

17. If $U = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$, $A = \{2, 4, 7\}$, $B = \{3, 5, 7, 9, 11\}$ and $C = \{7, 8, 9, 10, 11\}$, then compute $(A \cap U) \cap (B \cup C)$.
(a) $\{7\}$ (b) $\{8\}$ (c) $\{9\}$ (d) $\{6\}$

18. If A and B be any two sets, then $A \cap (A \cup B)'$ is equal to
(a) A (b) B
(c) ϕ (d) None of these

ASSERTION-REASON BASED QUESTIONS

In the following questions 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.
(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true but (R) is not the correct explanation of A.
(c) (A) is true but (R) is false.
(d) (A) is false but (R) is true.

19. **Assertion (A) :** If $A = \{x : x = 4n, n \in N\}$ and $B = \{x : x = 6n, n \in N\}$, then $A \cap B = \{24, 48, 72, 96...\}$
Reason (R) : $A \cap B = \{ln : n \in N$ and l = L.C.M. of $(4, 6)\}$.

20. **Assertion (A)** : If $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 4, 6\}$, $C = \{3, 4, 6\}$, then $(A \cup B) \cap C = \{3, 4, 6\}$.
Reason (R) : $(A \cup B)' = A' \cap B'$.

SECTION - B

This section comprises of very short answer type questions (VSA) of 2 marks each.

21. If $a \in N$ such that $aN = \{an : n \in N\}$, then describe the set $2N \cap 7N$.

OR

Let N be the universal set. Also, let
$A = \{x : x \in N$ and x is even$\}$
$B = \{x : x \in N$ and x is divisible by 2 and 3$\}$. Find A', B' and hence show that $(A')' = A$.

22. Let Z denote the set of all integers and $A = \{(a, b) : a^2 + 3b^2 = 28, a, b \in Z\}$ and $B = \{(a, b) : a > b, a, b \in Z\}$. Then find the number of elements in $A \cap B$.

23. If A and B are two sets such that $n(A \times B) = 6$ and some elements of $A \times B$ are $(-1, 2)$, $(2, 3)$, $(4, 3)$, then find $A \times B$ and $B \times A$.

OR

If A and B are two sets, then $A \subset B$ iff $B^c \subset A^c$.

24. Given that $N = \{1, 2, 3, ..., 100\}$. Then, write
(i) the subset of N whose elements are even numbers.
(ii) the subset of N whose elements are perfect square numbers.

25. For any sets A, B and C, using the properties of sets prove the following.

(i) $A - (B - C) = (A - B) \cup (A \cap C)$

(ii) $A \cup B = (A - B) \cup (A \cap B) \cup (B - A)$

SECTION - C

This section comprises of short answer type questions (SA) of 3 marks each.

26. If $U = \{x : x \in Z, -2 \le x \le 10\}$, $A = \{x : x = 2p + 1, p \in Z, -1 \le p \le 4\}$ and $B = \{x : x = 3q + 1, q \in Z, -1 \le q < 4\}$, then verify De-Morgan's laws for complementation.

27. Find the union of each of the following pairs of sets.

(i) $A = \{a, e, i, o, u\}$, $B = \{a, c, d\}$

(ii) $A = \{x : x$ is a natural number and $1 < x \le 5\}$ and $B = \{x : x$ is a natural number and $5 < x \le 10\}$

OR

Let $A = \{a, b, c, d\}$, $B = \{c, d, e, f\}$ and $C = \{e, f, g, h\}$. Verify the associative law of union of sets.

28. If $A = \left\{\frac{-1}{2}, 0, \frac{1}{4}, \frac{3}{4}, 2\right\}$, $B = \left\{0, \frac{1}{4}, \frac{3}{4}, 2, \frac{5}{2}\right\}$ and $C = \left\{\frac{-1}{2}, \frac{1}{4}, 1, 2, \frac{5}{2}\right\}$, then verify the associative property of intersection of sets.

29. If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{2, 4, 6, 8\}$ and $B = \{2, 3, 5, 7\}$, then find $(A \cup B)$, $(A \cap B)$ and $(A \Delta B)$.

30. Using Venn diagram, verify $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

OR

Using Venn diagram, verify $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

31. Let $U = \{1, 2, 3, 4, 5, 6, 8\}$, $A = \{2, 3, 4\}$, $B = \{3, 4, 5\}$. Show that $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$.

SECTION - D

This section comprises of long answer type questions (LA) of 5 marks each.

32. Set A comprises all three digit numbers that are multiples of 5, set B comprises all three-digit even numbers that are multiples of 6 and set C comprises all three digit numbers that are multiples of 4. Find the number of elements in (i) $A - B$ (ii) B - C (iii) $A - C$.

33. Let $U = \{x : x \le 10, x \in N\}$, $A = \{x : x$ is a prime number $< 10\}$, $B = \{3x : x \in N, x < 4\}$. Verify that $(A \cup B)' = A' \cap B'$. Also, represent $(A \cup B)'$ with the help of Venn diagram.

OR

Let $A = \{n \in N : \text{H.C.F.}\ (n, 45) = 1\}$ and Let $B = \{2k : k \in \{1, 2, \ldots, 100\}\}$. Then find the sum of all the elements of $A \cap B$.

34. If $A = \{2, 3, 4\}$, $B = \{3, 4, 5\}$, $C = \{2, 4, 5\}$, and $D = \{3, 5, 6\}$ then verify that $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$.

35. For any sets A, B and C using properties of sets, prove that $(A - B) \cap (A - C) = A - (B \cup C)$.

OR

For two non-empty sets A and B show that $A \cap (B - C) = (A \cap B) - (A \cap C)$.

SECTION - E

This section comprises of 3 case-study/passage-based questions of 4 marks each with sub-parts. The first two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.

36. Out of 100 students; 15 passed in English, 12 passed in Mathematics, 8 in Science, 6 in English and Mathematics, 7 in Mathematics and Science, 4 in English and Science; 4 in all the three passed. Based on the above information answer the following questions.

(i) Find the number of students passed in English and Mathematics but not in Science.

(ii) Find the number of students passed in Mathematics and Science but not in English.

(iii) Find the number of students passed in Mathematics only.

OR

Find the number of students passed in more than one subject.

37. In a group of 50 students, the number of students studying French, English, Sanskrit were found to be as follows :

French = 17, English = 13, Sanskrit = 15, French and English = 9, English and Sanskrit = 4, French and Sanskrit = 5, English, French and Sanskrit = 3.

Based on the above information answer the following questions.

(i) Find the number of students who study English and Sanskrit but not French.

(ii) Find the number of students who study French and Sanskrit but not English.

(iii) Find the number of students who study atleast one of the three languages.

OR

Find the number of students who study none of the three languages.

38. Given the sets $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{2, 3, 4\}$ and the equations $A \Delta X = A - B$ and $(A \Delta Y) \Delta B = A - B$.

Based on the above information answer the following questions.

(i) Find the set X. **(ii)** Find the set Y.

SOLUTIONS

1. (d) : Given that, $A = \{x : x = 4|n| + 1, -5 \le n \le -2, n \in Z\} = \{21, 17, 13, 9\}$

$\therefore$ Number of elements in A is 4.

So, number of subsets $= 2^4 = 16$

2. (b) **3. (d)**

4. (c) : $A = \{\phi, 1\}$

All subsets of A are $\phi, \{\phi\}, \{1\}, \{\phi, 1\}$.

5. (c) : $A = \{3, 6, 9, 12, ...\}, B = \{6, 12, 18, 24, ...\}$

$\therefore \quad A - B = \{3, 9, 15, 21, ...\}$.

6. (b) : $(A \cup B) \cap (A \cup C) = A \cup (B \cap C)$

$= \{2, 3, 4, 8, 10\} \cup \{4, 5, 12\} = \{2, 3, 4, 5, 8, 10, 12\}$

7. (b) : $A = \{3^1, 3^2, 3^3, 3^4, 3^5, 3^6\} = \{3, 9, 27, 81, 243, 729\}$

$B = \{9^1, 9^2, 9^3, 9^4\} = \{9, 81, 729, 6561\}$

$A \cap B = \{9, 81, 729\}$

8. (b)

9. (a) : Consider $(A \cup B')' \cap (A' \cup B)'$

$= (A' \cap B) \cap (A \cap B') = (B - A) \cap (A - B) = \phi$

10. (b) : $(A \cap B) \cup (A - B) = (A \cap B) \cup (A \cap B')$

$[\because (A - B) = (A \cap B')]$

$= A \cap (B \cup B')$ (By distributive law)

$= A \cap U = A$ $(\because B \cup B' = U)$

11. (b) : We can say that

$A \cap B = \{2, 4, 6, 8, 12, 20\} \cap \{3, 6, 9, 12, 15\} = \{6, 12\}$

$B \cap C = \{3, 6, 9, 12, 15\} \cap \{5, 10, 15, 20\} = \{15\}$

$C \cap A = \{5, 10, 15, 20\} \cap \{2, 4, 6, 8, 12, 20\} = \{20\}$

and $A \cap B \cap C = \phi$

From the given options only option (b) satisfies all the conditions.

12. (d) : There is no natural number less than 1

$\therefore$ Option (a) is an empty set.

There is no natural number between 3 and 4

$\therefore$ Option (b) is an empty set.

Between two consecutive integers, there exist no integer. So option (c) is an empty set.

Now, $x^2 = 2 \ \forall \ x \in R \Rightarrow x = \pm\sqrt{2}$

$\therefore \quad A = \{-\sqrt{2}, \sqrt{2}\}$, which contain two elements.

$\therefore$ It is not an empty set.

13. (a) : $B = \{x : x \in R, 2 \le x \le 3\}$

$\Rightarrow \quad x \in [2, 3]$ (Using meaning of $a \le x \le b$)

14. (d)

15. (b) : Given, $A = \{3, 6, 9, 12\}$

$= \{x : x = 3n, n \in N$ and $1 \le n \le 4\}$

and $B = \{1, 4, 9, ..., 100\} = \{x : x = n^2, n \in N$ and $1 \le n \le 10\}$

16. (a) : (a) $\{x : |x| < 1, x \in I\} = \{x : -1 < x < 1, x \in I\} = \{0\}$

(b) $\{x : |x| = 5, x \in I\} = \{x : x = \pm 5\} = \{\pm 5\}$

(c) $\{x : x^2 = 1, x \in I\} = \{x : x = \pm 1\} = \{\pm 1\}$

(d) $\{x : x^2 + x + 1 = 0, x \in R\}$

$$= \left\{x : x = \frac{-1 \pm \sqrt{3}i}{2}, x \in R\right\} = \phi$$

17. (a) : Here $A \cap U = \{2, 4, 7\}$;

$B \cup C = \{3, 5, 7, 8, 9, 10, 11\}$.

$\therefore \quad (A \cap U) \cap (B \cup C) = \{2, 4, 7\} \cap \{3, 5, 7, 8, 9, 10, 11\}$

$= \{7\}$

18. (c) : $A \cap (A \cup B)' = A \cap (A' \cap B') = (A \cap A') \cap B'$

$= \phi \cap B' = \phi$

19. (d) : $A = \{x : x = 4n, n \in N\} = \{4, 8, 12, 16, 20, 24, \ldots\}$

$B = \{x : x = 6n, n \in N\} = \{6, 12, 18, 24, 30, \ldots\}$

$\therefore \quad A \cap B = \{12, 24, 36, \ldots\}$

$= \{1 \times 12, 2 \times 12, 3 \times 12, 4 \times 12, \ldots\}$

$= \{ln : n \in N, l = \text{L.C.M. of } (4, 6) = 12\}$

$\therefore$ (A) is false but (R) is true.

20. (b) : Given, $A = \{1, 2, 3, 4, 5\}, B = \{2, 4, 6\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

$\therefore \quad (A \cup B) \cap C = \{1, 2, 3, 4, 5, 6\} \cap \{3, 4, 6\} = \{3, 4, 6\}$

Both (A) and (R) are true but (R) is not the correct explanation of (A).

21. We have, $aN = \{an : n \in N\}$

$\therefore \quad 2N = \{2n : n \in N\} = \{2, 4, 6, 8, 10, 12, 14, 16, ...\}$

and $7N = \{7n : n \in N\} = \{7, 14, 21, 28, ...\}$

Now, $2N \cap 7N = \{14, 28, 42, ...\} = \{14n : n \in N\} = 14N$

Note : $aN \cap bN = dN$, where d is the LCM of a and b.

OR

Clearly, $A = \{2, 4, 6, 8, ...\}$

and $B = \{x : x \in N$ and x is divisible by 6$\}$

$= \{6, 12, 18, ...\}$

Now, $A' = \{1, 3, 5, 7, 9, ...\}$ = set of odd numbers

and B' = Set of numbers which are not divisible by 6.

$= \{x : x \in N, x$ is not divisible by 6$\}$

Also, $(A')' = \{2, 4, 6, 8, ...\} = A$.

22. $A = \{(a, b) : a^2 + 3b^2 = 28, a, b \in Z\}$
(a, b) can be (1, 3), (–1, 3), (1, –3), (–1, –3), (5, 1), (–5, 1), (5, –1), (–5, –1), (4, 2) (–4, 2), (4, –2), (–4, –2)
$\therefore \quad n(A) = 12, n(B) = \infty$
So, $A \cap B = \{(a, b) : a^2 + 3b^2 = 28$ and $a > b, a, b \in Z\}$
$\Rightarrow \quad n(A \cap B) = 6$

23. It is given that $n(A \times B) = 6 = 3 \times 2 = n(A) \times n(B)$
So, $A = \{-1, 2, 4\}$ and $B = \{2, 3\}$.
$\therefore \quad A \times B = \{(-1, 2), (-1, 3), (2, 2), (2, 3), (4, 2), (4, 3)\}$
and $B \times A = \{(2, -1), (3, -1), (2, 2), (3, 2), (2, 4), (3, 4)\}$

OR

Let $A \subset B$, we shall prove that $B^c \subset A^c$.
For all $x \in B^c \Rightarrow x \notin B \Rightarrow x \notin A \Rightarrow x \in A^c \quad (\because A \subset B)$
Hence, $B^c \subset A^c$
Conversely, let $B^c \subset A^c$, we shall prove that $A \subset B$
For all $x \in A \Rightarrow x \notin A^c \Rightarrow x \notin B^c \Rightarrow x \in B$
Hence, $A \subset B$. Therefore, $A \subset B$ iff $B^c \subset A^c$.

24. We have given $N = \{1, 2, 3, 4, ..., 100\}$
(i) Let Y be the subset of N whose elements are even numbers, then
$X = \{2, 4, 6, ..., 100\}$
(ii) Let Y be the subset of N whose elements are perfect square numbers, then
$Y = \{1, 4, 9, 16, 25, 36, 49, 64, 81, 100\}$

25. **(i)** Consider L.H.S., $A - (B - C) = A - (B \cap C')$
$= A \cap (B \cap C')' = A \cap (B' \cup C)$
$= (A \cap B') \cup (A \cap C) = (A - B) \cup (A \cap C) =$ R.H.S.
(ii) Consider R.H.S., $(A - B) \cup (A \cap B) \cup (B - A)$
$= (A \cap B') \cup (A \cap B) \cup (B \cap A')$
$= A \cap (B' \cup B) \cup (B \cap A')$
$= (A \cap U) \cup (B \cap A') = A \cup (B \cap A') = (A \cup B) \cap (A \cup A')$
$= (A \cup B) \cap U = A \cup B =$ L.H.S.

26. Clearly, $U = \{-2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$,
$A = \{-1, 1, 3, 5, 7, 9\}$ and $B = \{-2, 1, 4, 7, 10\}$
(a) $(A \cup B)' = A' \cap B'$.
Here, $A \cup B = \{-2, -1, 1, 3, 4, 5, 7, 9, 10\}$
$\therefore \quad (A \cup B)' = \{0, 2, 6, 8\}$ ...(i)
Also, $A' = \{-2, 0, 2, 4, 6, 8, 10\}$
and $B' = \{-1, 0, 2, 3, 5, 6, 8, 9\}$
$\therefore \quad A' \cap B' = \{0, 2, 6, 8\}$ ...(ii)
From (i) and (ii), we get $(A \cup B)' = A' \cap B'$.
(b) $(A \cap B)' = A' \cup B'$
Here, $A \cap B = \{1, 7\}$
$\therefore \quad (A \cap B)' = \{-2, -1, 0, 2, 3, 4, 5, 6, 8, 9, 10\}$ ...(iii)
Also, $A' \cup B' = \{-2, -1, 0, 2, 3, 4, 5, 6, 8, 9, 10\}$ ...(iv)
From (iii) and (iv), we get $(A \cap B)' = A' \cup B'$.

27. **(i)** Given, $A = \{a, e, i, o, u\}$, $B = \{a, c, d\}$
$\Rightarrow \quad A \cup B = \{a, c, d, e, i, o, u\}$
(ii) Given, $A = \{x : x$ is a natural number and $1 < x \le 5\} \Rightarrow A = \{2, 3, 4, 5\}$
and $B = \{x : x$ is a natural number and $5 < x \le 10\}$
$\Rightarrow \quad B = \{6, 7, 8, 9, 10\}$
Now, $A \cup B = \{2, 3, 4, 5\} \cup \{6, 7, 8, 9, 10\}$
$= \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$

OR

Here, we need to verify
$A \cup (B \cup C) = (A \cup B) \cup C$
Clearly, $B \cup C = \{c, d, e, f, g, h\}$
$\therefore \quad A \cup (B \cup C) = \{a, b, c, d\} \cup \{c, d, e, f, g, h\}$
$= \{a, b, c, d, e, f, g, h\}$ ...(i)
and $A \cup B = \{a, b, c, d, e, f\}$
$\therefore \quad (A \cup B) \cup C = \{a, b, c, d, e, f\} \cup \{e, f, g, h\}$
$= \{a, b, c, d, e, f, g, h\}$ ...(ii)
From (i) and (ii), we get
$A \cup (B \cup C) = (A \cup B) \cup C$

28. Here, we need to verify the following
$A \cap (B \cap C) = (A \cap B) \cap C$

Clearly, $B \cap C = \left\{\frac{1}{4}, 2, \frac{5}{2}\right\}$ and $A \cap B = \left\{0, \frac{1}{4}, \frac{3}{4}, 2\right\}$

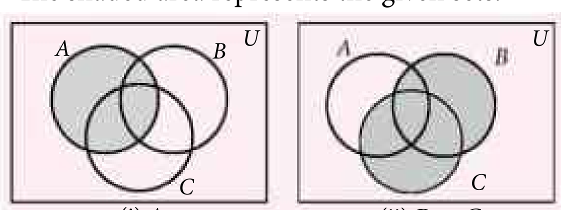
$$\therefore \ A \cap (B \cap C) = \left\{-\frac{1}{2}, 0, \frac{1}{4}, \frac{3}{4}, 2\right\} \cap \left\{\frac{1}{4}, 2, \frac{5}{2}\right\} = \left\{\frac{1}{4}, 2\right\} \quad ...(i)$$

$$\text{and } (A \cap B) \cap C = \left\{0, \frac{1}{4}, \frac{3}{4}, 2\right\} \cap \left\{-\frac{1}{2}, \frac{1}{4}, 1, 2, \frac{5}{2}\right\} = \left\{\frac{1}{4}, 2\right\} \quad ...(ii)$$

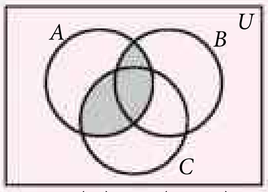
From (i) and (ii), we get $A \cap (B \cap C) = (A \cap B) \cap C$
Hence, the associative property of intersection of sets is verified.

29. Given sets are $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$,
$A = \{2, 4, 6, 8\}$ and $B = \{2, 3, 5, 7\}$
$A \cup B = \{2, 4, 6, 8\} \cup \{2, 3, 5, 7\} = \{2, 3, 4, 5, 6, 7, 8\}$
$A \cap B = \{2, 4, 6, 8\} \cap \{2, 3, 5, 7\} = \{2\}$
Now, $A - B = \{2, 4, 6, 8\} - \{2, 3, 5, 7\} = \{4, 6, 8\}$
and $B - A = \{2, 3, 5, 7\} - \{2, 4, 6, 8\} = \{3, 5, 7\}$
$\therefore \quad A \Delta B = (A - B) \cup (B - A) = \{3, 4, 5, 6, 7, 8\}$

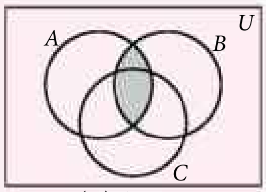
30. The shaded area represents the given sets.



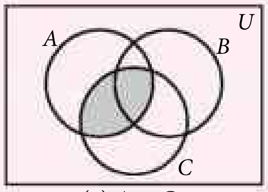
(i) A (ii) $B \cup C$



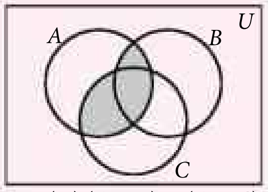
(iii) $A \cap (B \cup C)$



(iv) $A \cap B$



(v) $A \cap C$

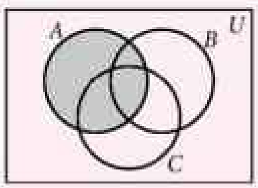


(vi) $(A \cap B) \cup (A \cap C)$

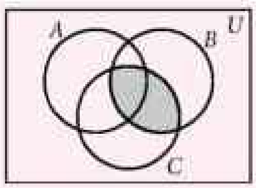
From (iii) and (vi), it is clear that
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

OR

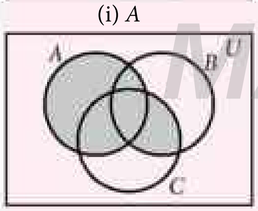
The shaded area represents the given sets.



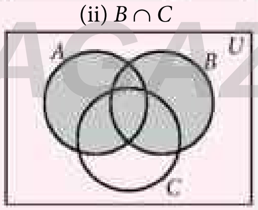
(i) A



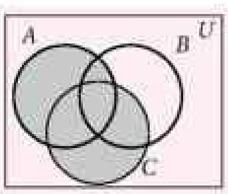
(ii) $B \cap C$



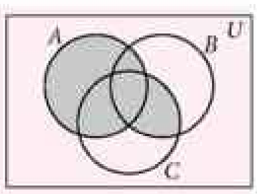
(iii) $A \cup (B \cap C)$



(iv) $A \cup B$



(v) $A \cup C$



(vi) $(A \cup B) \cap (A \cup C)$

From (iii) and (vi), it is clear that
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

31. Given $U = \{1, 2, 3, 4, 5, 6, 8\}$, $A = \{2, 3, 4\}$, $B = \{3, 4, 5\}$

$A \cup B = \{2, 3, 4\} \cup \{3, 4, 5\} = \{2, 3, 4, 5\}$

$\therefore \quad (A \cup B)' = \{2, 3, 4, 5\}' = \{1, 6, 8\}$ ...(i)

$A' = \{2, 3, 4\}' = \{1, 5, 6, 8\}$

$B' = \{3, 4, 5\}' = \{1, 2, 6, 8\}$

$\therefore \quad A' \cap B' = \{1, 5, 6, 8\} \cap \{1, 2, 6, 8\} = \{1, 6, 8\}$ ...(ii)

From (i) and (ii), we get

$(A \cup B)' = A' \cap B'$

Now, $A \cap B = \{2, 3, 4\} \cap \{3, 4, 5\} = \{3, 4\}$

$(A \cap B)' = \{1, 2, 5, 6, 8\}$ ...(iii)

$A' \cup B' = \{1, 5, 6, 8\} \cup \{1, 2, 6, 8\} = \{1, 2, 5, 6, 8\}$ ...(iv)

From (iii) and (iv), we get

$(A \cap B)' = A' \cup B'$.

32. Clearly, $A = \{100, 105, 110, ..., 995\}$ = Set of multiples of 5 from 100 to 999

$B = \{102, 108, 114, ..., 996\}$ = Set of multiples of 6 from 100 to 999

$C = \{100, 104, 108, ..., 996\}$ = Set of multiples of 4 from 100 to 999

$\therefore \quad n(A) = \frac{995}{5} - 19 = 180, \; n(B) = \frac{996}{6} - 16 = 150,$

$n(C) = \frac{996}{4} - 24 = 225$

$n(A \cap B)$ = Set of multiples of 30 = $\frac{990}{30} - 3 = 30$

$n(B \cap C)$ = Set of multiples of 12 = $\frac{996}{12} - 8 = 75$

$n(A \cap C)$ = Set of multiples of 20 = $\frac{980}{20} - 4 = 45$

(i) $n(A - B) = n(A) - n(A \cap B) = 180 - 30 = 150$

(ii) $n(B - C) = n(B) - n(B \cap C) = 150 - 75 = 75$

(iii) $n(A - C) = n(C) - n(A \cap C) = 225 - 45 = 180$

33. We have, $U = \{1, 2, 3, ..., 10\}$,

$A = \{2, 3, 5, 7\}, \quad B = \{3, 6, 9\}$

$A \cup B = \{2, 3, 5, 7\} \cup \{3, 6, 9\} = \{2, 3, 5, 6, 7, 9\}$

$(A \cup B)' = \{2, 3, 5, 6, 7, 9\}' = \{1, 4, 8, 10\}$ ...(i)

$A' = \{2, 3, 5, 7\}' = \{1, 4, 6, 8, 9, 10\}$

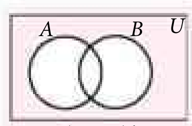
$B' = \{3, 6, 9\}' = \{1, 2, 4, 5, 7, 8, 10\}$

$A' \cap B' = \{1, 4, 6, 8, 9, 10\} \cap \{1, 2, 4, 5, 7, 8, 10\}$
$= \{1, 4, 8, 10\}$ ...(ii)

From (i) and (ii), we get

$(A \cup B)' = A' \cap B'$

Also, Venn diagram of $(A \cup B)' = A' \cap B'$ is as follows:



$(A \cup B)'$

OR

We have, $A = \{n \in N : \text{H.C.F}\,(n, 45) = 1\}$

$B = \{2K : K \in \{1, 2, ..., 100\}\}$

$B = \{2, 4, 6, 8, 10, 12, 14, 16, 18, ..., 200\}$

Let the subset of B, elements of which are divisible by 3.

$X = \{6, 12, 18, ..., 198\} \Rightarrow n(X) = 33$

Let the subset of B, elements of which are divisible by 5.

$Y = \{10, 20, 30, ..., 200\} \Rightarrow n(Y) = 20$.

Let the subset of B, elements of which are divisible by both 3 and 5.

$X \cap Y = \{30, 60, ..., 180\}$, $n(X \cap Y) = 6$
Required sum $= \Sigma B - [\Sigma X + \Sigma Y - \Sigma(X \cap Y)]$
$= 10100 - (3366 + 2100 - 630) = 5264$

34. Here, $A = \{2, 3, 4\}$, $B = \{3, 4, 5\}$, $C = \{2, 4, 5\}$ and $D = \{3, 5, 6\}$
Now, $(A \times B) = \{2, 3, 4\} \times \{3, 4, 5\}$
$= \{(2, 3), (2, 4), (2, 5), (3, 3), (3, 4), (3, 5), (4, 3), (4, 4), (4, 5)\}$
$(C \times D) = \{2, 4, 5\} \times \{3, 5, 6\}$
$= \{(2, 3), (2, 5), (2, 6), (4, 3), (4, 5), (4, 6), (5, 3), (5, 5), (5, 6)\}$
$\therefore (A \times B) \cap (C \times D) = \{(2, 3), (2, 5), (4, 3), (4, 5)\}$ ...(i)
Also, $A \cap C = \{2, 4\}$ and $B \cap D = \{3, 5\}$
$\therefore \quad (A \cap C) \times (B \times D) = \{2, 4\} \times \{3, 5\}$
$= \{(2, 3), (2, 5), (4, 3), (4, 5)\}$ ...(ii)
Hence, from (i) and (ii), we have
$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

35. Let $x \in (A - B) \cap (A - C)$ ...(i)
$\Rightarrow \quad x \in (A - B)$ and $x \in (A - C)$
$\Rightarrow \quad (x \in A$ and $x \notin B)$ and $(x \in A$ and $x \notin C)$
$\Rightarrow \quad x \in A$ and $(x \notin B$ and $x \notin C) \Rightarrow x \in A$ and $x \notin (B \cup C)$
$\Rightarrow \quad x \in A - (B \cup C)$ ...(ii)
Hence, from (i) and (ii),
$(A - B) \cap (A - C) \subset A - (B \cup C)$ ...(iii)
Now, let $y \in A - (B \cup C)$ ...(iv)
$\Rightarrow \quad y \in A$ and $y \notin (B \cup C) \Rightarrow y \in A$ and $(y \notin B$ and $y \notin C)$
$\Rightarrow \quad (y \in A$ and $y \notin B)$ and $(y \in A$ and $y \notin C)$
$\Rightarrow \quad y \in (A - B)$ and $y \in (A - C)$
$\Rightarrow \quad y \in (A - B) \cap (A - C)$ ...(v)
Hence, from (iv) and (v)
$A - (B \cup C) \subset (A - B) \cap (A - C)$ ...(vi)
From (iii) and (vi), we get
$A - (B \cup C) = (A - B) \cap (A - C)$

OR

Let x be any arbitrary element of $A \cap (B - C)$
Then, $x \in A \cap (B - C) \Rightarrow x \in A$ and $x \in (B - C)$
$\Rightarrow \quad x \in A$ and $(x \in B$ and $x \notin C)$
$\Rightarrow \quad (x \in A$ and $x \in B)$ and $(x \in A$ and $x \notin C)$
$\Rightarrow \quad x \in (A \cap B)$ and $x \notin (A \cap C)$
$\Rightarrow \quad x \in (A \cap B) - (A \cap C)$
$\Rightarrow \quad A \cap (B - C) \subseteq (A \cap B) - (A \cap C)$ ...(i)
Now, let y be any arbitrary element of $(A \cap B) - (A \cap C)$
Then, $y \in (A \cap B) - (A \cap C)$
$\Rightarrow \quad y \in (A \cap B)$ and $y \notin (A \cap C)$
$\Rightarrow \quad (y \in A$ and $y \in B)$ and $(y \in A$ and $y \notin C)$
$\Rightarrow \quad y \in A$ and $(y \in B$ and $y \notin C)$
$\Rightarrow \quad y \in A$ and $y \in B - C \Rightarrow y \in A \cap (B - C)$
$\Rightarrow \quad (A \cap B) - (A \cap C) \subseteq A \cap (B - C)$ ...(ii)
From (i) and (ii), we get
$A \cap (B - C) = (A \cap B) - (A \cap C)$

36. Let U, E, M and S denote the total number of students, number of students passed in English, Mathematics and Science, respectively.
Here, $n(U) = 100$, $n(E) = 15$, $n(M) = 12$, $n(S) = 8$, $n(E \cap M) = 6$, $n(M \cap S) = 7$, $n(E \cap S) = 4$ and $n(E \cap M \cap S) = 4$
(i) The number of students passed in English and Mathematics but not in Science
$= n(E \cap M \cap S') = n(E \cap M) - n(E \cap M \cap S) = 6 - 4 = 2$
(ii) The number of students passed in Mathematics and Science but not in English
$n(M \cap S \cap E') = n(M \cap S) - n(M \cap S \cap E) = 7 - 4 = 3$
(iii) The number of students passed in Mathematics only
$= n(M \cap E' \cap S')$
$= n(M) - n(M \cap E) - n(M \cap S) + n(M \cap E \cap S)$
$= 12 - 6 - 7 + 4 = 16 - 13 = 3$

OR

The number of students passed in more than one subject
$= n(M \cap E) + n(M \cap S) + n(S \cap E) - 2n(M \cap E \cap S)$
$= 6 + 7 + 4 - 2(4) = 17 - 8 = 9$

37. Let F be the set of students who study French, E be the set of students who study English and S be the set of students who study Sanskrit.

Given, $n(U) = 50$, $n(F) = 17$, $n(E) = 13$, and $n(S) = 15$
$n(F \cap E) = 9$, $n(E \cap S) = 4$, $n(F \cap S) = 5$
$n(F \cap E \cap S) = 3$
(i) $n(E \cap S \cap F') = n[(E \cap S) \cap F']$
$= n(E \cap S) - n(E \cap S \cap F) = 4 - 3 = 1$
(ii) $n(F \cap S \cap E') = n(F \cap S) - n(F \cap S \cap E) = 5 - 3 = 2$
(iii) $n(F \cup E \cup S) = n(F) + n(E) + n(S) - n(F \cap E) - n(E \cap S) - n(F \cap S) + n(F \cap E \cap S)$
$= 17 + 13 + 15 - 9 - 4 - 5 + 3 = 30$

OR

$n(F' \cap E' \cap S') = n(U) - n(F \cup E \cup S) = 50 - 30 = 20$

38. **(i)** We know that $A \Delta (A \Delta B) = B$
Now, $A \Delta X = A - B \Rightarrow A \Delta(A \Delta X) = A \Delta (A - B)$
$\Rightarrow \quad X = A \Delta (A - B)$
But $A - B = \{1, 5, 6, 7\}$, $A - (A - B)$
$= \{2, 3, 4\}$, $(A - B) - A = \phi$
Consequently, $X = A \Delta (A - B)$
$= (A - (A - B)) \cup ((A - B) - A) = A - (A - B) = \{2, 3, 4\} = B.$
(ii) $(A \Delta Y) \Delta B = A - B \Leftrightarrow (A \Delta B) \Delta Y = A - B$
$\Leftrightarrow \quad (A \Delta B) \Delta ((A \Delta B) \Delta Y) = (A \Delta B) \Delta (A - B)$
$\Leftrightarrow \quad Y = (A \Delta B) \Delta (A - B)$
Now, $A \Delta B = (A - B) \cup (B - A) = \{1, 5, 6, 7\} \cup \phi$
$= \{1, 5, 6, 7\}$. Thus, $Y = (A \Delta B) \Delta (A - B) = \phi$

MONTHLY TEST DRIVE

XI

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 10 : Statistics and Probability

Time Taken : 60 Min.

Only One Option Correct Type

1. The mean of the data set comprising of 16 observations is 16. If one of the observation valued 16 is deleted and three new observations valued 3, 4 and 5 are added to the data, then the mean of the resultant data, is

(a) 15.8 (b) 14.0 (c) 16.8 (d) 16

2. The sum of 100 observations and the sum of their squares are 400 and 2475, respectively. Later on, three observations, 3, 4 and 5 were found to be incorrect. If the incorrect observations are omitted, then the variance of the remaining observations is

(a) 8.00 (b) 8.25 (c) 9.00 (d) 8.50

3. Two different families A and B are blessed with equal number of children. There are 3 tickets to be distributed amongst the children of these families so that no child gets more than one ticket. If the probability that all the tickets go to the children of the family B is 1/12, then the number of children in each family is

(a) 3 (b) 5 (c) 4 (d) 6

4. For three events A, B and C,
P(Exactly one of A or B occurs)
= P(Exactly one of B or C occurs)
= P(Exactly one of C or A occurs) = 1/4
and P(All the three events occur simultaneously) = 1/16. Then the probability that at least one of the events occurs, is

(a) 7/16 (b) 7/64 (c) 3/16 (d) 7/32

5. Select the correct statement.

(a) If the probability that a person visiting a zoo will see the giraffe is 0.72 and the probability that he will see the bears is 0.84. Then the probability that he will see both is 0.52.

(b) The possible probabilities that a typist will make 0, 1, 2, 3, 4, 5 or more mistakes in typing a report are, respectively, 0.12, 0.25, 0.36, 0.14, 0.08 and 0.11.

(c) The probability of intersection of two events A and B is always less than or equal to those favourable to the event A.

(d) None of these.

6. Four candidates A, B, C and D have applied for the assignment to coach a school cricket team. If A is twice as likely to be selected as B and B and C are given about the same chance of being selected, while C is twice as likely to be selected as D, what is the probability that A will not be selected?

(a) 4/9 (b) 5/9 (c) 9/4 (d) 9/5

One or More Than One Option(s) Correct Type

7. A boy has a collection of blue and green marbles. The number of blue marbles belong to the sets {2, 3, 4, ... 13}. If two marbles are chosen simultaneously and at random from his collection, then the probability that they have different colour is 1/2. Possible number of blue marbles is :

(a) 2 (b) 3 (c) 6 (d) 10

8. For two given events A and B, $P(A \cap B)$

(a) not less than $P(A) + P(B) - 1$

(b) not greater than $P(A) + P(B)$

(c) equal to $P(A) + P(B) - P(A \cup B)$

(d) equal to $P(A) + P(B) + P(A \cup B)$

9. If M and N are two events, the probability that exactly one of them occurs is

(a) $P(M) + P(N) - 2P(M \cap N)$

(b) $P(M) + P(N) - P(\overline{M \cup N})$

(c) $P(\bar{M}) + P(\bar{N}) - 2P(\bar{M} \cap \bar{N})$

(d) $P(M \cap \bar{N}) - P(\bar{M} \cap N)$

10. E and F are two independent events. The probability that both E and F happen is 1/12 and the probability that neither E nor F happens is 1/2, then
(a) $P(E) = 1/3, P(F) = 1/4$
(b) $P(E) = 1/2, P(F) = 1/6$
(c) $P(E) = 1/6, P(F) = 1/2$
(d) $P(E) = 1/4, P(F) = 1/3$

11. Mean of the numbers 1, 2, 3, ..., n with respective weights $1^2 + 1, 2^2 + 2, 3^2 + 3, ..., n^2 + n$ is
(a) $\frac{3n(n+1)}{2(2n+1)}$ (b) $\frac{3n^2+7n+2}{2(2n+4)}$
(c) $(3n + 1)/4$ (d) $(3n + 1)/2$

12. The variable x takes two values x_1 and x_2 with frequencies f_1 and f_2, respectively. If σ denotes the standard deviation of x, then
(a) $\sigma^2 = \frac{f_1x_1^2 + f_2x_2^2}{f_1 + f_2} - \left(\frac{f_1x_1 + f_2x_2}{f_1 + f_2}\right)^2$
(b) $\sigma^2 = \frac{f_1f_2}{(f_1 + f_2)^2}(x_1 - x_2)^2$
(c) $\sigma^2 = \frac{(x_1 - x_2)^2}{f_1 + f_2}$ (d) None of these

13. For the expansion of $(1 + x)^{30}$, which of the following is CORRECT?
(a) The mean of coefficients is $2^{30}/31$
(b) The median of coefficients is ${}^{30}C_{15}$.
(c) The median of coefficients is the average of 15th and 16th term.
(d) None of these

Comprehension Type

Paragraph for Q. No. 14 and 15

Two fair dice are rolled. Let $P(A_i) > 0$ denotes the probability of the event that the sum of the number appearing on the faces of the dice is divisible by i.

14. Which one of the following events is most probable?
(a) A_3 (b) A_4 (c) A_5 (d) A_6

15. Which one of the following pairs (i, j) the events A_i and A_j are independent?
(a) (3, 4) (b) (4, 6) (c) (2, 3) (d) (4, 2)

Matrix Match Type

16. If runs of two players A and B in cricket matches are such that player A has mean 150 and variance 100 and player B has mean 143 and variance 225 of runs.
Match the following measures in Column I with their corresponding values in Column II.

Column I		Column II	
P.	Coefficient of variation of player A	(1)	A
Q.	Coefficient of variation of player B	(2)	B
R.	The more consistent player is	(3)	10.48
S.	Less consistent player is	(4)	6.67

(a) P-2, Q-1, R-4, S-3 (b) P-4, Q-3, R-1, S-2
(c) P-3, Q-4, R-2, S-1 (d) P-2, Q-4, R-1, S-3

Numerical Answer Type

17. The mean square deviation of set of observations $x_1, x_2, x_3, \ldots, x_n$ about a point c is defined as $\frac{1}{n}\sum_{i=1}^{n}(x_i - c)^2$. If mean square deviation about –1 and 1 of a set of observations are 7 and 3 respectively, then the standard deviation of these observations is _______.

18. Let A, B, C be three events. If the probability of occurring exactly one event out of A and B is $1 - a$, out B and C is $1 - 2a$, out of C and A is $1 - a$ and that of occurring three events simultaneously is a^2. If the probability that atleast one out of A, B, C will occur is greater than λ, then the value of 2050 λ must be _____.

19. The number lock of a suitcase has 4 wheels, each labelled with ten digits, *i.e.*, from 0 to 9. The lock opens with a sequence of four digits with no repeats. The probability of a person getting the right sequence to open the suitcase is p, then the value of $5040p$ is ____.

20. In an experiment with 15 observations of x, the following results were available.
$\sum x^2 = 2830, \ \sum x = 170$
One observation that was 20 was found to be wrong and was replaced by the correct value 30. Then the corrected variance is ________.

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ......
No. of questions correct ......
Marks scored in percentage ......

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

CBSE warm-up!

CLASS-XII

Chapterwise practice questions for CBSE Exams as per the latest pattern and reduced syllabus by CBSE for the academic session 2023-24.

Series-1 Relations and Functions

Time Allowed : 3 hours
Maximum Marks : 80

General Instructions

(a) This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
(b) Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
(c) Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
(d) Section C has 6 Short Answer (SA)-type questions of 3 marks each.
(e) Section D has 4 Long Answer (LA)-type questions of 5 marks each.
(f) Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION - A (MULTIPLE CHOICE QUESTIONS)

Each question carries 1 mark.

1. If $A = \{a, b, c\}$ and $B = \{-2, -1, 0, 1, 2\}$, then total number of one-one functions from A to B is
(a) 60 (b) 30 (c) 15 (d) 20

2. Which one of the following relations on R is an equivalence relation?
(a) $aR_1b \Rightarrow |a| = |b|$ (b) $aR_2b \Rightarrow a \geq b$
(c) $aR_3b \Rightarrow a$ divides b (d) $aR_4b \Rightarrow a < b$

3. A Signum Function $f : R \to R$ defined by

$$f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases} \text{ is}$$

(a) one-one (b) onto
(c) many one (d) none of these

4. Consider a non-empty set consisting of children of family and relation R defined as aRb if a is brother of b. Then R is
(a) symmetric but not transitive
(b) transitive but not Symmetric
(c) neither symmetric nor transitive
(d) both symmetric and transitive

5. If $f : R \to R$ be a function defined by $f(x) = \dfrac{x^2 - 8}{x^2 + 2}$, then f is
(a) one-one but not onto
(b) one-one and onto
(c) onto but not one-one
(d) neither one-one nor onto

6. Let set A has 7 elements and set B has 8 elements, then number of one-one mapping that can be defined from A to B is
(a) 56 (b) 5760 (c) 40320 (d) 192

7. If $n(A) = n(B) = 3$, then how many bijections from A to B can be formed?
(a) 5 (b) 3 (c) 6 (d) 2

8. If $f : R \to B$ given by $f(x) = \sin x$ is onto function, then the set B is equal to
(a) $[-1, \infty)$ (b) $(0, \infty)$
(c) $[-1, 0]$ (d) $[-1, 1]$

9. Let $X = \{-1, 0, 1\}$, $Y = \{0, 2\}$ and a function $f: X \to Y$ defined by $y = 2x^4$, is
(a) one-one onto (b) one-one into
(c) many-one onto (d) many-one into

10. If $f: R \to R$ is given by
$$f(x)=\begin{cases}-1, \text{ when } x \text{ is rational,}\\ 1, \text{ when } x \text{ is irrational}\end{cases}$$
then $(fof)(1-\sqrt{3})$ is equal to
(a) 1 (b) −1 (c) $\sqrt{3}$ (d) 0

11. Given $f(x)=\log\left(\frac{1+x}{1-x}\right)$ and $g(x)=\frac{3x+x^3}{1+3x^2}$, then $fog(x)$ equals
(a) $-f(x)$ (b) $3f(x)$
(c) $[f(x)]^3$ (d) none of these

12. Let L denote the set of all straight lines in a plane. Let a relation R be defined by $\alpha R \beta \Leftrightarrow \alpha \perp \beta$, $\alpha, \beta \in L$. Then, R is
(a) reflexive only (b) symmetric only
(c) transitive only (d) none of these

13. Let $A = \{1, 2, 3, 4\}$ and R be a relation in A given by $R = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1), (3, 1)\}$. Then, R is
(a) reflexive
(b) transitive
(c) an equivalence relation
(d) none of these

14. Let $P = \{(x, y) : x^2 + y^2 = 1, x, y \in R\}$. Then, P is
(a) reflexive (b) symmetric
(c) transitive (d) anti-symmetric

15. The function $f: R \to R$ given by $f(x) = x^3 - 1$ is
(a) one-one but not onto
(b) onto but not one-one
(c) a bijection
(d) neither one-one nor onto

16. The number of bijective functions from set A to itself when A contains 106 elements is
(a) 106 (b) $(106)^2$ (c) $(106)!$ (d) 2^{106}

17. Let R be a relation on the set N of natural numbers denoted by $nRm \Leftrightarrow n$ is a factor of m (*i.e.*, $n \mid m$). Then, R is
(a) reflexive and symmetric
(b) transitive and symmetric
(c) equivalence
(d) reflexive, transitive but not symmetric

18. If $f: [1, \infty) \to [2, \infty)$ is given by $f(x)=x+\frac{1}{x}$, then $f^{-1}(x)$ equals
(a) $\frac{x+\sqrt{x^2-4}}{2}$ (b) $\frac{x}{1+x^2}$
(c) $\frac{x-\sqrt{x^2-4}}{2}$ (d) $1+\sqrt{x^2-4}$

ASSERTION-REASON BASED QUESTIONS

In the following questions 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true but (R) is not the correct explanation of A.
(c) (A) is true but (R) is false.
(d) (A) is false but (R) is true.

19. **Assertion (A) :** If set A contains 7 elements and set B contains 6 elements, then the number of one-one onto mapping from A to B is 420.
Reason (R) : If A and B are two non-empty sets containing m and n elements respectively, then number of one-one onto functions from A to B
$$=\begin{cases}n!, \text{ if } m=n\\ 0, \text{ if } m\neq n\end{cases}.$$

20. **Assertion (A) :** If R is a relation defined on the set of natural numbers N such that $R = \{ (x, y) : x, y \in N \text{ and } 2x + y = 24 \}$, then R is not an equivalence relation.
Reason (R) : A relation is said to be an equivalence relation if it is reflexive, symmetric but not transitive.

SECTION - B

This section comprises of very short answer type questions (VSA) of 2 marks each.

21. Let the relation R be defined in N by aRb, if $2a + 3b = 30$. Find R.

OR

Let R be the equivalence relation in the set $A = \{0, 1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : 2 \text{ divides } (a - b)\}$. Write the equivalence class [0].

22. Let f and g be functions from R to R defined as

$$f(x)=\begin{cases}7x^2+x-8, x\le 1\\ 4x+5, 1<x\le 7\\ 8x+3, x>7\end{cases} \text{ and } g(x)=\begin{cases}|x|, x<-3\\ 0, -3\le x<2\\ x^2+4, x\ge 2\end{cases}$$

Then, find $(fog)(-3)$ and $(fog)(9)$.

23. Let A be any non-empty set, then prove that identify function on set A is a bijection.

24. State the reason for the relation R in the set {1, 2, 3} given by $R = \{(1, 2), (2, 1)\}$ not to be transitive.

OR

Let $A = \{1, 2, 3, 4\}$ and $B = \{5, 6, 7, 8, 9\}$ and set $f = \{(1, 5), (2, 6), (3, 7), (4, 8)\}$ be a function from A to B. State whether f is one-one or not.

25. State whether the function $f: [-1, 1] \to R$ given by $f(x)=\frac{x}{x+2}, x\ne -2$ is one-one or not?

SECTION - C

This section comprises of short answer type questions (SA) of 3 marks each.

26. Let R be a relation defined on the set of natural numbers N as follows :
$R = \{(x, y) : x \in N, y \in N, 2x + y = 41\}$. Check whether, R is reflexive, symmetric and transitive.

OR

Let R be a relation on the set N be defined by $\{(x, y) : x, y \in N, 3x + y = 43\}$. Then show that R is none of reflexive, symmetric and transitive.

27. Let $f : R \to R$ be defined by (i) $f(x) = x + |x|$ (ii) $f(x) = x + 1$. Determine whether f is onto or not.

28. Show that $f : R^+ \to R^+$ defined by $f(x)=\frac{1}{2x}$ is bijective, where R^+ is the set of all non zero positive real number.

OR

Let N be the set of natural numbers and relation R on set N be defined by $R = \{(x, y) : x, y \in N, x + 4y = 10\}$. Check whether R is reflexive, symmetric and transitive.

29. Let $A=\left\{x : x\in R, -\frac{\pi}{2}\le x\le\frac{\pi}{2}\right\}$ and $B = \{y : y \in R, -1\le y\le 1\}$. Show that the function $f: A\to B$ such that $f(x) = \sin x$ is bijective.

30. Find the inverse of the function $y=\frac{10^x-10^{-x}}{10^x+10^{-x}}$.

31. Prove that the relation R in the set of natural numbers N be defined as $R = \{a^2 - 4ab + 3b^2 = 0, a, b \in N\}$ is reflexive but neither symmetric nor transitive.

SECTION - D

This section comprises of long answer type questions (LA) of 5 marks each.

32. m is said to be related to n if m and n are integers and $m - n$ is divisible by 13. Does this define an equivalence relation?

33. Let N denotes the set of all natural numbers and R be the relation on $N \times N$ defined by $(a, b)\ R\ (c, d) \Leftrightarrow ad\ (b + c) = bc\ (a + d)$. Check whether, R is an equivalence relation on $N \times N$.

OR

On the set Z of all integers, consider the relation $R = \{(a, b) : (a - b)$ is divisible by 3}
Show that R is an equivalence relation on Z.
Also, find the partitioning of Z into mutually disjoint equivalence class.

34. Show that the function $f : R \to R$ defined by $x^3 + x$ is a bijection.

OR

Given a function defined by $f(x)=\sqrt{4-x^2}$, $0 \le x \le 2$, $0 \le f(x) \le 2$. Show that f is bijective function.

35. If $f : Q - \{3\} \to Q$ be a function defined by $f(x)=\frac{2x+3}{x-3}$, show that f is one-one but not onto.

SECTION - E

This section comprises of 3 case-study/passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub parts of 2 marks each.

36. Sherlin and Danju are playing Ludo at home. While rolling the dice, Sherlin's sister Raji observed and noted the possible outcomes of the throw every time belongs to set {1, 2, 3, 4, 5, 6}. Let A be the set of players while B be the set of all possible outcomes.

$A = \{S, D\}$, $B = \{1, 2, 3, 4, 5, 6\}$

Based on the above information, answer the following questions.

(i) Let $R : B \to B$ be defined by $R = \{(x, y) : y$ is divisible by $x\}$. Show that R is reflexive and transitive but not symmetric.

(ii) Raji wants to know the number of functions from A to B. How many number of functions are possible?

(iii) Let R be a relation on B defined by $R = \{(1, 2), (2, 2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5)\}$. Check whether the given relation is equivalence or not?

OR

Raji wants to know the number of relations possible from A to B. How many number of relations are possible?

37. Read the following passage and answer the questions given below.

Consider the mapping $f : A \to B$ defined by

$f(x) = \dfrac{x-1}{x-2}$ such that f is a bijection.

(i) Find the domain of f.

(ii) If $g : R - \{2\} \to R - \{1\}$ is defined by $g(x) = 2f(x) - 1$, then find $g(x)$ in terms of x.

(iii) Find the range of f.

OR

Which type of function is g defined in part (ii)?

38. Four friends A, B, C and D were playing Ludo. The rolling of dice indicated the possible outcomes of throw every time which is related to the set $\{1, 2, 3, 4, 5, 6\}$. If we consider P as the set of players while Q as the set of all possible outcomes. Based on the above information, answer the following questions.

(i) One of the friends wanted to know the number of one-one functions from P to Q. How many number of one-one functions are possible?

(ii) How many onto functions are possible from Q to P ?

SOLUTIONS

1. (a) : Let $f : A \to B$ be a one-one function.

$\therefore$ Since, $n(A) = 3 = r$ and $n(B) = 5 = q$

$\therefore$ Total number of one-one functions

$= {}^qP_r = {}^5P_3 = 60$

2. (a) : Reflexive : $a \in R$, $aR_1\, a \Rightarrow |a| = |a|$

$\therefore$ R_1 is reflexive.

Symmetric : $a, b \in R$, then $aR_1\, b \Rightarrow |a| = |b|$

$\Rightarrow |b| = |a| \Rightarrow bR_1\, a$

$\therefore$ R_1 is symmetric

Transitive : Let $a, b \in R$, then $aR_1\, b \Rightarrow |a| = |b|$ ...(i)

and $bR_1 c \Rightarrow |b| = |c|$ ...(ii)

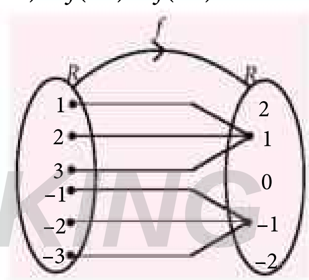
From (i) and (ii), $|a| = |c| \Rightarrow aR_1\, c$

$\therefore$ R_1 is transitive

$\Rightarrow$ R_1 is an equivalence relation.

3. (c) : We have, $f(1) = f(2) = f(3) = 1$;

$f(0) = 0$ and $f(-1) = f(-2) = f(-3) = -1$



Since, f is not one-one.

$\therefore$ Signum Function is many one.

4. (b) : Given that a non-empty set consisting of children in a family and a relation R defined as aRb if a is brother of b. Here aRb is not symmetric in case of brother and sister as if aRb satisfies but bRa does not satisfy.

Hence, it is transitive but not symmetric.

5. (d) : Given function $f : R \to R$ defined by

$$f(x) = \frac{x^2 - 8}{x^2 + 2}$$

We observe that for negative and positive values of x, we get same value of $f(x)$.

Hence, $f(x)$ is not one-one.

Also, $y = f(x) \Rightarrow y = \dfrac{x^2-8}{x^2+2} \Rightarrow yx^2 + 2y = x^2 - 8$

$$\Rightarrow \; x = \sqrt{\frac{2y+8}{1-y}}$$

For $y = 1$, we can't define x, hence f is not onto.

6. (c) : We know that if A and B are two non-empty sets containing m and n elements respectively, then number of one-one function from A to B = $\begin{cases} {}^nP_m, \text{ if } n \geq m \\ 0, \text{ if } n = m \end{cases}$

Here, $n = 8$ and $m = 7$

$\therefore$ Required number of one-one mapping = ${}^8P_7$

$= \frac{8!}{(8-7)!} = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40320$

7. (c) : We know that if A and B are two non-empty sets containing m and n elements respectively, then number of bijections from A to B = $\begin{cases} n!, \text{ if } m = n \\ 0, \text{ if } m \neq n \end{cases}$

Here $m = n = 3$

$\therefore$ Required number of bijections = 3! = 3 × 2 = 6.

8. (d) : Given, $f(x) = \sin x \ \forall x \in R$

As, $\sin x \in [-1, 1]$ $\quad \therefore \quad f(x) \in [-1, 1]$

As f is onto

$\therefore \quad B = \{x : x \in [-1, 1]\}$

9. (c) : We have, $y = 2x^4$

$\therefore \quad y(-1) = y(1) = 2, y(0) = 0$ (many-one onto)

Here, we see that for two different values of x, we will get a same image and no element of y is left, which do not have pre-image. So, function is many-one onto.

10. (b) : $(fof)(1-\sqrt{3}) = f[f(1-\sqrt{3})] = f(1) = -1$

11. (b) : $f[g(x)] = f\left[\frac{3x+x^3}{1+3x^2}\right] = \log\left\{\frac{1+\frac{3x+x^3}{1+3x^2}}{1-\frac{3x+x^3}{1+3x^2}}\right\}$

$\Rightarrow \quad f[g(x)] = \log\left(\frac{1+x}{1-x}\right)^3 = 3\log\left(\frac{1+x}{1-x}\right) = 3f(x)$

12. (b) : Given, $\alpha R \beta \Leftrightarrow \alpha \perp \beta \ \therefore \ \alpha \perp \beta \Leftrightarrow \beta \perp \alpha$

$\Rightarrow \quad \beta R \alpha$

Hence, R is symmetric.

13. (a) : Reflexive : (1, 1), (2, 2), (3, 3), (4, 4) $\in R$;

$\therefore \quad R$ is reflexive.

14. (b) : The relation is not reflexive and transitive but it is symmetric, because

$x^2 + y^2 = 1 \ \Rightarrow \ y^2 + x^2 = 1$

15. (c) : Given, $f(x) = x^3 - 1$. Let $x_1, x_2 \in R$.

Now, $f(x_1) = f(x_2) \quad \Rightarrow \quad x_1^3 - 1 = x_2^3 - 1 \quad \Rightarrow \quad x_1 = x_2$

$\therefore \quad f(x)$ is one-one. Also, it is onto.

Hence, it is a bijection.

16. (c) : The total number of bijections from a set containing n elements to itself is $n!$

Hence, required number = (106)!

17. (d) : Reflexive : $n \mid n$ for all $n \in N \Rightarrow R$ is reflexive.

Symmetric : $2 \mid 6$ but 6 does not divides 2 $\Rightarrow R$ is not symmetric.

Transitive : Let nRm and $mRp \Rightarrow n \mid m$ and $m \mid p$

$\Rightarrow \quad n \mid p \ \Rightarrow \ nRp$. So, R is transitive.

18. (a) : Let $y = f(x) = x + \frac{1}{x}$

$\Rightarrow \quad x^2 - xy + 1 = 0$

$\Rightarrow \quad x = \frac{y \pm \sqrt{y^2 - 4}}{2}$

$\because \quad y \in [2, \infty) \ \Rightarrow \ x \in [1, \infty)$

Hence, $f^{-1}(x) = \frac{x + \sqrt{x^2 - 4}}{2}$

19. (d) : Clearly, Reason is true.

Now, $m = 7$ and $n = 6$ *i.e.*, $m \neq n$

$\therefore$ Number of one-one onto mapping from A to B is 0.

$\therefore$ (A) is false but (R) is true.

20. (c) : (A) is true but (R) is false.

21. Given, $2a + 3b = 30 \Rightarrow 3b = 30 - 2a \Rightarrow b = \frac{30 - 2a}{3}$

Now, we choose the numbers $a \in N$, so that $b \in N$.

For $a = 3$, $b = 8$; For $a = 6$, $b = 6$; For $a = 9$, $b = 4$; For $a = 12$, $b = 2$

Hence, $R = \{(3, 8), (6, 6), (9, 4), (12, 2)\}$

OR

Here, $R = \{(a, b) \in A \times A : 2 \text{ divides } (a - b)\}$

This is an equivalence relation, where $A = \{0, 1, 2, 3, 4, 5\}$.

$\therefore \quad [0] = \{0, 2, 4\}$

22. We have, $g(-3) = 0$

$\therefore \quad fog(-3) = f(g(-3)) = f(0) = 7(0)^2 + 0 - 8 = -8$

Now, $g(9) = 9^2 + 4 = 85$

$\Rightarrow \quad fog(9) = f(g(9)) = f(85) = 8 \times 85 + 3 = 683$

23. The identify function $I_A : A \to A$ is defined as

$I_A(x) = x \ \forall \ x \in A$

Let $x_1, x_2 \in A$, then $I_A(x_1) = I_A(x_2) \Rightarrow x_1 = x_2$

$\therefore \quad I_A$ is one-one or injective

Let $y \in A$ be any arbitrary element, then there exist $x = y \in A$ such that, $I_A(x) = x = y$

$\therefore \quad I_A$ is onto or surjective.

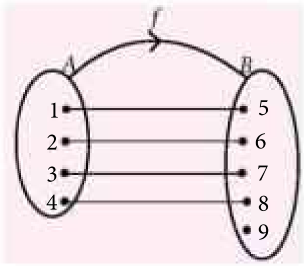
Hence, I_A is bijective function.

24. We have, $R = \{(1, 2), (2, 1)\}$ defined on set $\{1, 2, 3\}$.

As, $(1, 2) \in R$, $(2, 1) \in R$ but $(1, 1) \notin R$.

$\therefore \quad R$ is not transitive.

OR



Since, every element of A has its unique image in B.

$\therefore$ f is one-one.

25. Let $x_1, x_2 \in [-1, 1]$ such that $f(x_1) = f(x_2)$

$$\Rightarrow \frac{x_1}{x_1+2} = \frac{x_2}{x_2+2}$$

$\Rightarrow$ $x_1x_2 + 2x_1 = x_1x_2 + 2x_2 \Rightarrow x_1 = x_2$

Hence, f is one-one.

26. Given, $R = \{(x, y) : x \in N, y \in N, 2x + y = 41\}$

$R = \{(1, 39), (2, 37), (3, 35), ..., (19, 3), (20, 1)\}$

Reflexive : $(3, 3) \notin R$ as $2 \times 3 + 3 \neq 41$ $\therefore$ R is not reflexive

Symmetric : As $(1, 39) \in R$ but $(39, 1) \notin R$

$\therefore$ R is not symmetric

Transitive : As $(11, 19) \in R$ and $(19, 3) \in R$ but $(11, 3) \notin R$

$\therefore$ R is not transitive

$\therefore$ R is neither reflexive, nor symmetric and nor transitive.

OR

Given, $R = \{(x, y) : x, y \in N, 3x + y = 43\}$

Reflexive : $(1, 1) \notin R$ as $3 \cdot 1 + 1 = 4 \neq 43$

So, R is not reflexive.

Symmetric : $(1, 40) \in R$ but $(40, 1) \notin R$. So R is not symmetric.

Transitive : $(14, 1) \in R$ and $(1, 40) \in R$. But $(14, 40) \notin R$. So, R is not transitive.

27. (i) $f(x) = x + |x| = \begin{cases} x+x, \text{ if } x \geq 0 \\ x-x, \text{ if } x < 0 \end{cases} = \begin{cases} 2x, \text{ if } x \geq 0 \\ 0, \text{ if } x < 0 \end{cases}$

Thus, $f(x) = 2x \geq 0 \ \forall \ x \geq 0$ and $f(x) = 0 \ \forall \ x < 0$

$\therefore$ $f(x)$ can't be negative for any $x \in R$

Thus, f is not onto.

(ii) We have $f(x) = x + 1 \ \forall \ x \in R$

For any $y \in R$, $y = f(x) \Rightarrow y = x + 1 \Rightarrow x = y - 1$

$\therefore$ $f(y - 1) = y - 1 + 1 = y$

Hence, f is onto.

28. Here, $f : R^+ \to R^+$ defined by $f(x) = \dfrac{1}{2x}$

One-One : let $x_1, x_2 \in R^+$ (domain)

Now, $f(x_1) = f(x_2) \Rightarrow \dfrac{1}{2x_1} = \dfrac{1}{2x_2}$

$\Rightarrow$ $2x_1 = 2x_2 \Rightarrow x_1 = x_2$

$\therefore$ f is one-one

Onto : Let $y \in R^+$ (co-domain) be any arbitrary element, then $y \neq 0$

Let $y = f(x) \Rightarrow y = \dfrac{1}{2x} \Rightarrow x = \dfrac{1}{2y} \in R^+$

$\therefore$ f is onto. Hence, f is bijective where $\dfrac{1}{2y}$ is non zero real number. Hence, each element of co-dominan (R^+) is the image of some element of domain (R^+).

OR

We have, $R = \{(x, y) : x, y \in N, x + 4y = 10\}$

$\therefore$ $R = \{(2, 2), (6, 1)\}$

Reflexive : Let $x \in N$ be any element.

Since $(x, x) \notin R$, $\therefore$ R is not reflexive.

Symmetric : Since $(6, 1) \in R$ but $(1, 6) \notin R$

$\therefore$ R is not symmetric.

Transitive : Let $x, y, z \in N$, then $(x, y) \in R$ and $(y, z) \in R$

$(x, y) \in R \Rightarrow x + 4y = 10$ ... (i)

and $(y, z) \in R \Rightarrow y + 4z = 10$ ... (ii)

From (i) and (ii), $x + 4(10 - 4z) = 10$

$\Rightarrow$ $x + 40 - 16z = 10 \Rightarrow x - 16z = -30$

$\therefore$ $(x, z) \notin R$

Thus, R is none of reflexive, symmetric and transitive.

29. Here, $A = \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$ and $B = [-1, 1]$

Also $f : A \to B$ such that $f(x) = \sin x$

$\therefore$ f is one-one.

$\because$ $f(x_1) = f(x_2) \Rightarrow \sin x_1 = \sin x_2$

$\Rightarrow$ $x_1 = x_2$ $\quad \left[\because x_1, x_2 \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]\right]$

Also, range $(f) = [-1, 1] = B$ so, f is onto.

Thus, f is one-one and onto and hence bijective.

30. Since, $y = \dfrac{10^{2x} - 1}{10^{2x} + 1} \Rightarrow y10^{2x} + y = 10^{2x} - 1$

$\Rightarrow$ $10^{2x} = \dfrac{1+y}{1-y}$ $\therefore$ $2x \log_{10} 10 = \log_{10} \dfrac{1+y}{1-y}$

$\Rightarrow$ $x = \dfrac{1}{2} \log_{10} \left(\dfrac{1+y}{1-y}\right)$ $\quad (\because \log_a a = 1)$

Hence, the inverse of y is $\dfrac{1}{2} \log_{10} \left(\dfrac{1+x}{1-x}\right)$.

31. $R = \{a^2 - 4ab + 3b^2 = 0, (a, b) \in N \times N\}$

Reflexive : Let $a \in N$

$\Rightarrow$ $a^2 - 4a^2 + 3a^2 = 0$ *i.e.*, $(a, a) \in R$ $\therefore$ R is reflexive.

Symmetric : Let $(3, 1) \in R \Rightarrow (3)^2 - 4(3)(1) + 3(1)^2 = 0$ but $(1, 3) \notin R$ as $(1)^2 - 4(1)(3) + 3(3)^2 \neq 0$

Thus, $(3, 1) \in R$ does not implies $(1, 3) \in R$

$\therefore$ R is not symmetric.

Transitive : Let $(9, 3) \in R$ and $(3, 1) \in R$

$\Rightarrow$ $9^2 - 4(9)(3) + 3(3)^2 = 81 - 108 + 27 = 0$ and

$3^2 - 4(3)(1) + 3(1)^2 = 9 - 12 + 3 = 0$

But $(9, 1) \notin R$ as $9^2 - 4(9)(1) + 3(1)^2 = 48 \neq 0$

$\therefore$ R is not transitive.

32. Z be the set of all integers.

$R = \{(m, n) : m - n$ is divisible by $13\}$

Reflexive : Let $m \in Z$

$m - m = 0 \Rightarrow m - m$ is divisible by 13

$\Rightarrow$ $(m, m) \in R$. $\therefore$ R is reflexive.

Symmetric : Let $m, n \in Z$ and $(m, n) \in R$

$\Rightarrow$ $m - n = 13p$ for some $p \in Z$

$\Rightarrow$ $n - m = 13(-p)$ where $(-p) \in Z$

$\Rightarrow$ $n - m$ is divisible by 13 $\Rightarrow (n, m) \in R$

$\therefore$ R is symmetric.

Transitive : Let $(m, n) \in R$ and $(n, q) \in R$ for some $m, n, q \in Z$

$\Rightarrow$ $m - n = 13p$ and $n - q = 13s$ for some $p, s \in Z$

$\Rightarrow$ $m - q = 13(p + s)$ $[\because p, s \in Z \Rightarrow p + s \in Z]$

$\Rightarrow$ $m - q$ is divisible by 13 $\Rightarrow (m, q) \in R$

$\therefore$ R is transitive.

Hence, R is an equivalence relation on Z.

33. Reflexive : Let (a, b) be an arbitrary element of $N \times N$. Then, $(a, b) \in N \times N \Rightarrow ab(b + a) = ba(a + b)$

[By commutativity of addition and multiplication on N]

$\Rightarrow$ $(a, b)\, R\, (a, b)$ for all $(a, b) \in N \times N$.

$\therefore$ R is reflexive on $N \times N$.

Symmetric : Let $(a, b), (c, d) \in N \times N$ be such that $(a, b)\, R\, (c, d)$.

$\Rightarrow$ $ad(b + c) = bc(a + d) \Rightarrow cb(d + a) = da(c + b)$

$\Rightarrow$ $(c, d)\, R\, (a, b)$

[By commutativity of addition and multiplication on N]

$\Rightarrow$ $(c, d)\, R\, (a, b)$

Thus, $(a, b)\, R\, (c, d)$

$\Rightarrow$ $(c, d)\, R\, (a, b)\ \forall\ (a, b), (c, d) \in N \times N$

$\therefore$ R is symmetric on $N \times N$.

Transitive: Let $(a,b),(c,d),(e,f) \in N \times N$ such that $(a,b)\,R\,(c,d)$ and $(c, d)\, R\, (e, f)$. Then, $(a, b)\, R\, (c, d)$

$\Rightarrow ad(b + c) = bc(a + d)$

$$\Rightarrow \frac{b+c}{bc} = \frac{a+d}{ad} \Rightarrow \frac{1}{b} + \frac{1}{c} = \frac{1}{a} + \frac{1}{d} \quad ...(i)$$

and $(c, d)\, R\, (e, f) \Rightarrow cf(d + e) = de(c + f)$

$$\Rightarrow \frac{d+e}{de} = \frac{c+f}{cf} \Rightarrow \frac{1}{d} + \frac{1}{e} = \frac{1}{c} + \frac{1}{f} \quad ...(ii)$$

Adding (i) and (ii), we get

$$\left(\frac{1}{b} + \frac{1}{c}\right) + \left(\frac{1}{d} + \frac{1}{e}\right) = \left(\frac{1}{a} + \frac{1}{d}\right) + \left(\frac{1}{c} + \frac{1}{f}\right)$$

$$\Rightarrow \frac{1}{b} + \frac{1}{e} = \frac{1}{a} + \frac{1}{f} = \frac{b+e}{be} = \frac{a+f}{af}$$

$\Rightarrow$ $af(b + e) = be(a + f) \Rightarrow (a, b)\, R\, (e, f)$

Thus, $(a, b)\, R\, (c, d)$ and $(c, d)\, R\, (e, f)$

$\Rightarrow$ $(a, b)\, R\, (e, f)$ for all $(a, b), (c, d), (e, f) \in N \times N$

$\therefore$ R is transitive on $N \times N$

Hence, R is an equivalence relation on $N \times N$.

OR

Z be a set of all integers and $R = \{(a, b) : (a - b)$ is divisible by $3\}$.

Reflexive : Let $a \in Z$, then $(a - a) = 0$, which is divisible by 3.

$\therefore$ $(a, a) \in R\ \forall\ a \in Z \therefore R$ is reflexive.

Symmetric : Let $a, b \in Z$ such that $(a, b) \in R$ then, $(a - b)$ is divisible by 3.

$\Rightarrow$ $(b - a)$ is divisible by $3 \Rightarrow (b, a) \in R \therefore (a, b) \in R$

$\Rightarrow$ $(b, a) \in R \therefore R$ is symmetric.

Transitive : Let $a, b, c \in Z$ such that $(a, b) \in R$ and $(b, c) \in R$

then, $(a, b) \in R \Rightarrow (a - b)$ is divisible by 3 and $(b, c) \in R$

$\Rightarrow$ $(b - c)$ is divisible by 3

$\Rightarrow$ $[(a - b) + (b - c)]$ is divisible by 3

$\Rightarrow$ $(a - c)$ is divisible by 3

Thus, $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow$ $(a, c) \in R$ for all $a, b, c \in Z$

Hence, R is an equivalence relation on Z

Let us consider [0], [1] and [2]

We have, $[0] = \{x \in Z : x R 0\}$

$= \{x \in Z : (x - 0)$ is divisible by $3\}$

$= \{...., -6, -3, 0, 3, 6, 9, ...\}$

Similarly, $[1] = \{x \in Z : x R 1\}$

$= \{x \in Z : (x - 1)$ is divisible by $3\} = \{...., -5, -2, 1, 4, 7, 10, ...\}$

Also, $[2] = \{x \in Z : x R 2\}$

$= \{x \in Z : (x - 2)$ is divisible by $3\} = \{...., -4, -1, 2, 5, 8, 11....\}$

Clearly [0], [1] and [2] are mutually disjoint and $Z = [0] \cup [1] \cup [2]$

34. We have $f : R \to R$ defined by $f(x) = x^3 + x$

Let $x_1, x_2 \in R$ such that $f(x_1) = f(x_2)$

$\Rightarrow$ $x_1^3 + x_1 = x_2^3 + x_2 \Rightarrow x_1^3 - x_2^3 + x_1 - x_2 = 0$

$\Rightarrow$ $(x_1 - x_2)(x_1^2 + x_1x_2 + x_2^2 + 1) = 0 \Rightarrow x_1 - x_2 = 0$

$[\because x_1^2 + x_1x_2 + x_2^2 \geq 0$ for all $x_1, x_2 \in R$.

$\therefore$ $x_1^2 + x_1x_2 + x_2^2 + 1 \geq 1$ for all $x_1, x_2 \in R] \Rightarrow x_1 = x_2$

$\Rightarrow$ f is one-one

Let y be any arbitrary element of R, then there exist $x \in R$ such that $f(x) = y$.

$\Rightarrow \quad x^3 + x = y \Rightarrow x^3 + x - y = 0$

Since, odd degree equation has atleast one real root.

Thus, for every value of y, the equation

$x^3 + x - y = 0$ has a real root α, such that $\alpha^3 + \alpha - y = 0$

$\Rightarrow \quad f(\alpha) = y$

Thus, for every $y \in R$, $\exists\ \alpha \in R$ such that $f(\alpha) = y$

So, f is onto function. Hence, $f: R \to R$ is a bijection.

OR

Let $y = f(x) = \sqrt{4 - x^2}, 0 \le x \le 2, 0 \le y \le 2$

Let x_1, x_2 be any two element in the interval $0 \le x \le 2$ such that $f(x_1) = f(x_2)$

$\Rightarrow \quad \sqrt{4 - x_1^2} = \sqrt{4 - x_2^2} \Rightarrow 4 - x_1^2 = 4 - x_2^2$

$\Rightarrow \quad x_1^2 = x_2^2 \Rightarrow x_1 = x_2$

[$\because x_1, x_2$ are non negative integer]

$\therefore \quad f(x)$ is one-one

Let $y \in [0, 2]$ be any arbitrary element, then there exists $x \in [0, 2]$ such that

$f(x) = y \Rightarrow \sqrt{4 - x^2} = y \Rightarrow 4 - x^2 = y^2 \Rightarrow x^2 = 4 - y^2$

$\Rightarrow \quad x = \pm\sqrt{4 - y^2} \Rightarrow x = \sqrt{4 - y^2}$

[$\because x$ is non negative integer]

Also, for $0 \le y \le 2$, we have

$0 \le \sqrt{4 - y^2} \le 2 \Rightarrow 0 \le x \le 2$, which is true.

Thus, for each $y \in [0, 2]$, $\exists\ x = \sqrt{4 - y^2} \in [0, 2]$ such that $f(x) = y$ $\therefore f(x)$ is onto

Hence, f is bijective function.

35. We have, $f: Q - \{3\} \to Q$ defined by $f(x) = \dfrac{2x + 3}{x - 3}$

Let $x_1, x_2 \in Q - \{3\}$ such that $f(x_1) = f(x_2)$

$\Rightarrow \quad \dfrac{2x_1 + 3}{x_1 - 3} = \dfrac{2x_2 + 3}{x_2 - 3}$

$\Rightarrow \quad 2x_1x_2 - 6x_1 + 3x_2 - 9 = 2x_1x_2 + 3x_1 - 6x_2 - 9$

$\Rightarrow \quad -6x_1 + 3x_2 - 3x_1 + 6x_2 = 0 \Rightarrow -9(x_1 - x_2) = 0$

$\Rightarrow \quad x_1 = x_2$

$\therefore \quad f$ is one-one

Let $y \in Q$ be arbitrary element, then there exist $x \in Q - \{3\}$

such that $f(x) = y \Rightarrow \dfrac{2x + 3}{x - 3} = y$

$\Rightarrow \quad 2x + 3 = xy - 3y \Rightarrow x(2 - y) = -3(y + 1)$

$\Rightarrow \quad x = \dfrac{-3(y + 1)}{2 - y} \notin Q - \{3\}$ for $y = 2$

$\therefore \quad f$ is not onto.

36. (i) $R = \{(x, y) : y$ is divisible by $x\}$

$= \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (1, 2), (1, 3), (1, 4), (2, 4), (1, 5), (1, 6), (2, 6), (3, 6)\}$

Here, $(x, x) \in R\ \forall\ x \in B \Rightarrow R$ is reflexive

$(1, 2) \in R$ but $(2, 1) \notin R \Rightarrow R$ is not symmetric

Clearly, R is transitive also.

(ii) Here, $n(A) = 2$ and $n(B) = 6$

$\therefore \quad$ Total number of functions from A to $B = 6^2 = 6 \times 6$

(iii) R is not reflexive as $(1, 1), (3, 3), (4, 4), (6, 6) \notin R$.

R is not symmetric as $(1, 2) \in R$ but $(2, 1) \notin R$.

R is not transitive as $(1, 3) \in R$ and $(3, 4) \in R$ but $(1, 4) \notin R$.

$\therefore \quad R$ is not an equivalence relation.

OR

We have, $n(A) = 2, n(B) = 6$

No. of relations from A to $B = 2^{n(A) \times n(B)} = 2^{2 \times 6} = 2^{12}$

37. (i) For $f(x)$ to be defined $x - 2 \ne 0$ *i.e.*, $x \ne 2$

$\therefore \quad$ Domain of $f = R - \{2\}$

(ii) We have, $g(x) = 2f(x) - 1$

$$= 2\left(\frac{x - 1}{x - 2}\right) - 1 = \frac{2x - 2 - x + 2}{x - 2} = \frac{x}{x - 2}$$

(iii) Let $y = f(x)$, then $y = \dfrac{x - 1}{x - 2}$

$\Rightarrow \quad xy - 2y = x - 1 \Rightarrow xy - x = 2y - 1 \Rightarrow x = \dfrac{2y - 1}{y - 1}$

Since, $x \in R - \{2\}$, therefore $y \ne 1$

Hence, range of $f = R - \{1\}$

OR

We have, $g(x) = \dfrac{x}{x - 2}$

Let $g(x_1) = g(x_2) \Rightarrow \dfrac{x_1}{x_1 - 2} = \dfrac{x_2}{x_2 - 2}$

$\Rightarrow \quad x_1x_2 - 2x_1 = x_1x_2 - 2x_2 \Rightarrow 2x_1 = 2x_2 \Rightarrow x_1 = x_2$

Thus, $g(x_1) = g(x_2) \Rightarrow x_1 = x_2$

Hence, $g(x)$ is one-one.

38. (i) Since, $P = \{A, B, C, D\}$ and $Q = \{1, 2, 3, 4, 5, 6\}$

$\therefore \quad n(P) = 4$ and $n(Q) = 6$

Thus, number of one-one functions from P to $Q = {}^6P_4$

$= \dfrac{6!}{(6 - 4)!} = 6 \times 5 \times 4 \times 3 = 360$

(ii) The number of onto functions from Q to P

$= \sum_{r=1}^{n} (-1)^{n-r} \cdot {}^nC_r\, r^m = \sum_{r=1}^{4} (-1)^{4-r} \cdot {}^4C_r\, r^6$

[Here $n = 4$ and $m = 6$]

$= (-1)^3 \cdot {}^4C_1(1)^6 + (-1)^2 \cdot {}^4C_2(2)^6 + (-1)^1 \cdot {}^4C_3(3)^6 + (-1)^0 \cdot {}^4C_4(4)^6$

$= -4 + 6(2)^6 - 4(3)^6 + (4)^6$

$= -4 + 384 - 2916 + 4096 = 1560$

MONTHLY TEST DRIVE XII

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 10 : Probability

Time Taken : 60 Min.

Only One Option Correct Type

1. A box contains N coins, of which m are fair and the rest are biased. The probability of getting head when a fair coin is tossed is 1/2, while it is 2/3 when a biased coin is tossed. A coin is drawn from the box at random and is tossed twice. The first time it shows head and the second time it shows tail. The probability that the coin drawn is fair, is
   (a) $\frac{5m}{m+8N}$ (b) $\frac{3m}{m+8N}$
   (c) $\frac{7m}{m+8N}$ (d) $\frac{9m}{m+8N}$

2. Let A and B be two events with $P(A^c) = 0.3$, $P(B) = 0.4$ and $P(A \cap B^c) = 0.5$. Then $P(B/A \cup B^c)$ is equal to
   (a) 1/4 (b) 1/3 (c) 1/2 (d) 2/3

3. A bag contains $2n + 1$ coins. It is known that n of these coins have a head on both sides, whereas the remaining $n + 1$ coins are fair. A coin is picked up at random from the bag and tossed. If the probability that the toss results in a head is 31/42, then n =
   (a) 10 (b) 11 (c) 12 (d) 13

4. A card is drawn and replaced in an ordinary pack of 52 playing cards. Minimum number of times must a card be drawn so that there is atleast an even chance of drawing a heart, is
   (a) 2 (b) 3
   (c) 4 (d) more than four

5. It is observed that 25% of the cases related to child labour reported to the police station are solved. If 6 new cases are reported, then the probability that atleast 5 of them will be solved is
   (a) $\left(\frac{1}{4}\right)^6$ (b) $\frac{19}{1024}$ (c) $\frac{19}{2048}$ (d) $\frac{19}{4096}$

6. Find the variance and standard deviation of the following probability distribution respectively. where $p + q = 1$

x_i	a	b
p_i	p	q

   (a) $ap + bq$, $\sqrt{ap+bq}$
   (b) $pq(a - b)^2$, $ap - bq$
   (c) $pq(a - b)^2$, $|a - b|\sqrt{pq}$
   (d) $\sqrt{ap-bq}$, $|a-b|\sqrt{pq}$

One or More Than One Option(s) Correct Type

7. If E and F are independent events such that $0 < P(E) < 1$ and $0 < P(F) < 1$ then
   (a) E and F are mutually exclusive
   (b) E and F^c are independent
   (c) E^c and F^c are independent
   (d) $P(E/F) + P(E^c/F) = 1$.

8. The p.d.f. of a continuous r.v. X is

$$f(x) = \begin{cases} \frac{1}{2a}, & 0 < x < 2a \ (a > 0) \\ 0, & \text{otherwise} \end{cases}, \text{ then}$$

   (a) $P\left(X < \frac{a}{2}\right) = P\left(X > \frac{3a}{2}\right)$
   (b) $P\left(X < \frac{a}{2}\right) < P\left(X > \frac{3a}{2}\right)$
   (c) $P\left(X < \frac{a}{2}\right) > P\left(X > \frac{3a}{2}\right)$
   (d) $P\left(X < \frac{a}{2}\right) = 2P\left(X > \frac{3a}{2}\right)$

9. A ball is drawn at random from box I and transferred to box II. If the probability of drawing a red ball from box I, after this transfer, is 1/3, then the correct option(s) with the possible values of n_1 and n_2 is (are)

(a) $n_1 = 4$ and $n_2 = 6$ (b) $n_1 = 2$ and $n_2 = 3$
(c) $n_1 = 10$ and $n_2 = 20$ (d) $n_1 = 3$ and $n_2 = 6$

10. Four persons A, B, C and D throw an unbiased die, turn by turn, in succession till one gets an even number and win the game. What is the probability that A wins the game if A begins?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{7}{15}$ (d) $\frac{8}{15}$

11. There are three bags B_1, B_2 and B_3. The bag B_1 contains 5 red and 5 green balls, B_2 contains 3 red and 5 green balls, and B_3 contains 5 red and 3 green balls. Bags B_1, B_2 and B_3 have probabilities 3/10, 3/10 and 4/10 respectively of being chosen. A bag is selected at random and a ball is chosen at random from the bag. Then which of the following options is/are correct?

(a) Probability that the chosen ball is green, given that the selected bag is B_3, equals 3/8.
(b) Probability that the selected bag is B_3, given that the chosen ball is green, equals 5/13.
(c) Probability that the chosen ball is green equals 39/80
(d) Probability that the selected bag is B_3, and the chosen ball is green equals 3/10

12. Let E and F be two independent events. The probability that exactly one of them occurs is 11/25 and the probability of none of them occurring is 2/25. If $P(T)$ denotes the probability of occurrence of the event T, then

(a) $P(E)=\frac{4}{5}, P(F)=\frac{3}{5}$ (b) $P(E)=\frac{1}{5}, P(F)=\frac{2}{5}$

(c) $P(E)=\frac{2}{5}, P(F)=\frac{1}{5}$ (d) $P(E)=\frac{3}{5}, P(F)=\frac{4}{5}$

13. The probabilities that a student passes in Mathematics, Physics and Chemistry are m, p and c respectively of these subjects, the student has a 75% chance of passing in at least one, a 50% chance of passing in atleast two, and a 40% chance of passing in exactly two. Which of the following relations are true.

(a) $p + m + c = 19/20$ (b) $p + m + c = 27/20$
(c) $pmc = 1/10$ (d) $pmc = 1/4$

Comprehension Type

Paragraph for Q. No. 14 and 15

Football teams T_1 and T_2 have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probabilities of T_1 winning, drawing and losing a game against T_2 are $\frac{1}{2}, \frac{1}{6}$ and $\frac{1}{3}$ respectively. Each team gets 3 points for a win, 1 point for a draw and 0 point for a loss in a game. Let X and Y denote the total points scored by teams T_1 and T_2 respectively, after two games.

14. $P(X > Y)$ is

(a) 1/4 (b) 5/12 (c) 1/2 (d) 7/12

15. $P(X = Y)$ is

(a) 11/36 (b) 1/3 (c) 13/36 (d) 1/2

Matrix Match Type

16. A bag contains 14 balls which are either white or black balls (all number of white and black balls are equally likely. Five balls are drawn at random from the bag without replacement. Match the column.

	Column I		Column II
(P)	Probability that all the five balls are black is equal to	(1)	3/13
(Q)	If the bag contains 11 black and 3 white balls, then the probability that all five balls are black is equal to	(2)	1/6
(R)	If all the five balls are black then the probability that the bag contains 11 black and 3 white balls is equal to	(3)	6/65
(S)	Probability that three balls are black and two are white is equal to	(4)	3/65

	P	Q	R	S		P	Q	R	S
(a)	2	3	2	1	(b)	2	1	3	2
(c)	1	2	3	1	(d)	1	2	3	2

Recipe for Success

"Life is like a bicycle. To keep your balance, you must keep moving"

– *Albert Einstein*

Numerical Answer Type

17. The life in hours of a radio tube is a c.r.v. X with

p.d.f. $f(x) = \begin{cases} \dfrac{100}{x^2}, & x \geq 100 \\ 0, & \text{elsewhere} \end{cases}$

Then the probability that a tube will last less than 200 hours if it is known that the tube is still functioning after 150 hours of service is ______.

18. A student appeared in an examination consisting of 8 true-false type questions. The student guesses the answers with equal probability. The smallest value of n, so that the probability of guessing at least 'n' correct answers is less than 1/2, is ______.

19. The probability that a missile hits a target successfully is 0.75. In order to destroy the target completely, at least three successful hits are required. Then the minimum number of missiles that have to be fired so that the probability of completely destroying the target is NOT less than 0.95, is _____

20. Let X be random variable with distribution.

x	-2	-1	3	4	6
$P(X = x)$	$\frac{1}{5}$	a	$\frac{1}{3}$	$\frac{1}{5}$	b

If the mean of X is 2.3 and variance of X is σ^2, then $100\,\sigma^2$ is equal to ______ .

◆◆

Keys are published in this issue. Search now! ☺

SELF CHECK

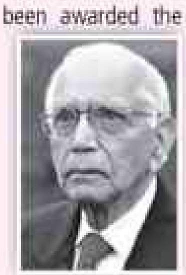
No. of questions attempted ......
No. of questions correct ......
Marks scored in percentage ......

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

Indian-American C.R. Rao wins Noblel Prize equivalent in statistics at the age of 102

The Indian-American statistician Calyampudi Radhakrishan Rao has been awarded the 2023 International Prize in Statistics - the equivalent of the Nobel prize for statistics. It is awarded once every two years to an individual or team "for major achievements using statistics to advance science, technology and human welfare".



Dr. C.R. Rao

The work of Professor Rao, 102, has influenced, in the words of the American Statistical Association, "not just statistics" but also "economics, genetics, anthropology, geology, national planning, demography, biometry and medicine".

The citation for his new award calls him "a professor whose work more than 75 years ago continues to exert a profound influence on science".

Professor Rao's ground breaking paper, "Information and accuracy attainable in the estimation of statistical parameters", was published in 1945. It was an impressive achievement since he was only 25 at the time. He would go on to do his Ph.D. in 1946-1948 at King's College, Cambridge University, under the supervision of Ronald Fisher, regraded as the father of modern statistics.

The 1945 paper boosted the development of modern statistics and its application in research.

Rao score test

One of Professor Rao's papers in 1948 offered a novel generic approach to testing hypotheses, now widely known as the "Rao score test". This and two other tests, developed by Jerzy Neyman, E.S. Pearson and Abraham Wald, are sometimes called "the hold trinity" of this branch of statistics.

Professor Rao also contributed to orthogonal arrays, a concept in combinatorics used to design experiments whose results are qualitatively good, as early as 1949.

A 19-year-old Rao couldn't secure a scholarship at Andhra University for administrative reasons. He was also rejected for a job at any Army survey unit. When he was staying in Calcutta, a chance meeting led him to enroll at the Indian Statistical Institute, where the spent the next four decades. After his retirement in 1979, he settled in the U.S.

The first half of the 20th century was the golden period of statistical theory in general, and Rao is one of the reasons for this being the case. As the renowned statistician Erich Lehmann wrote, Rao was "the person who did the most to continue [P.C.] Mahalanobis's work as a leader of statistics in India."

Courtesy : The Hindu